Subgrid-scale modeling for turbulent flow simulation with a Remeshed Vortex Method

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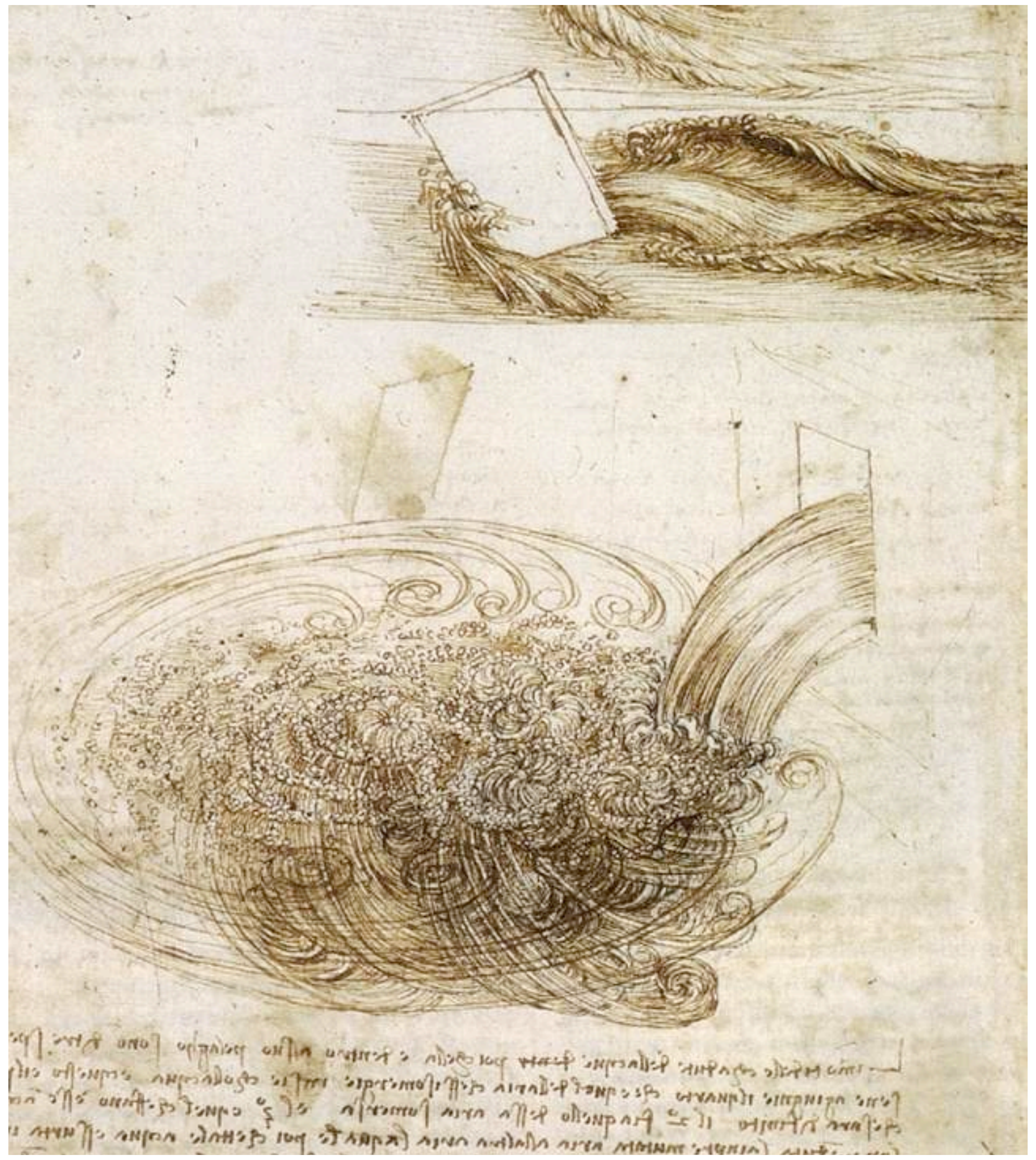
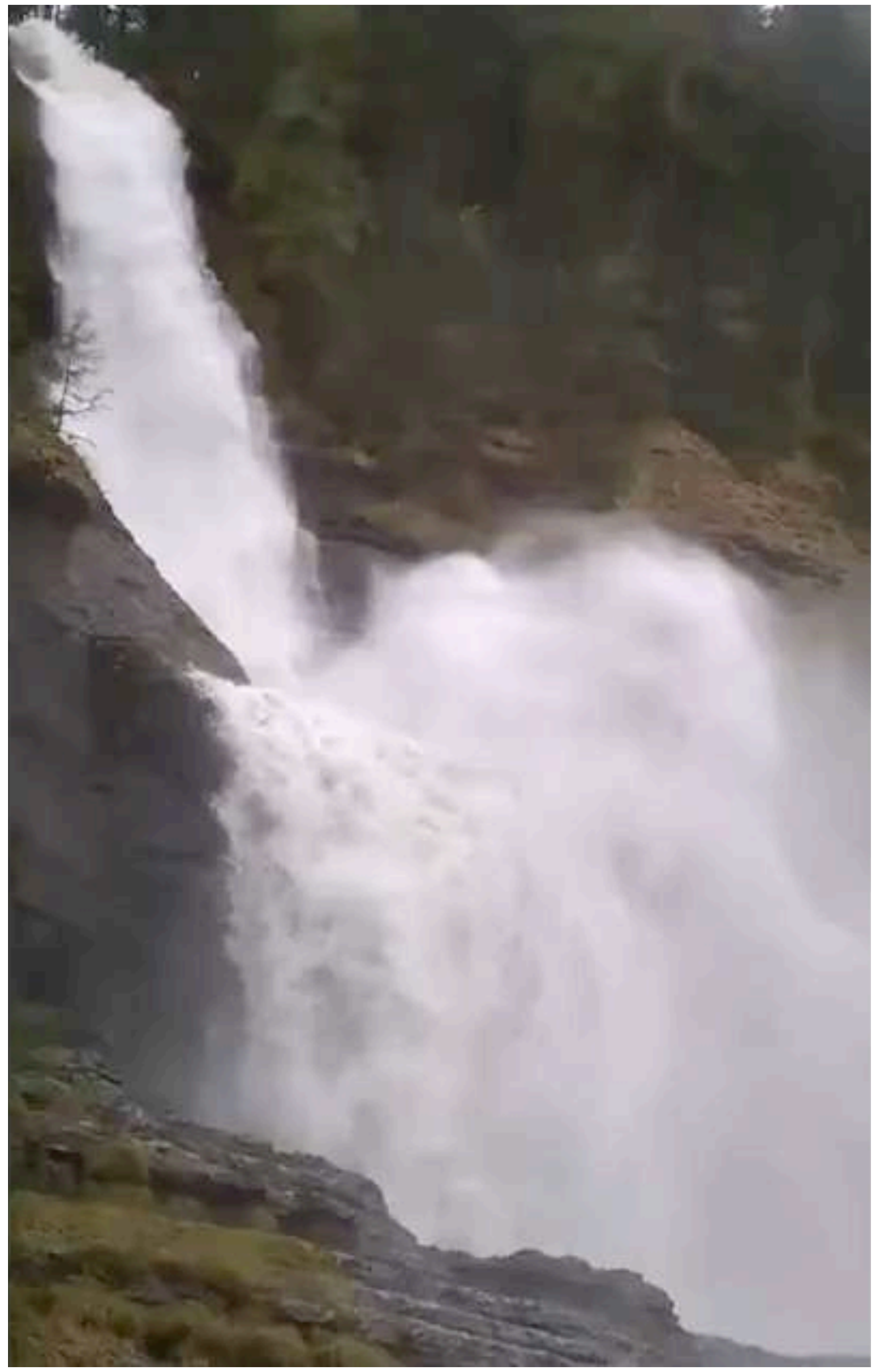
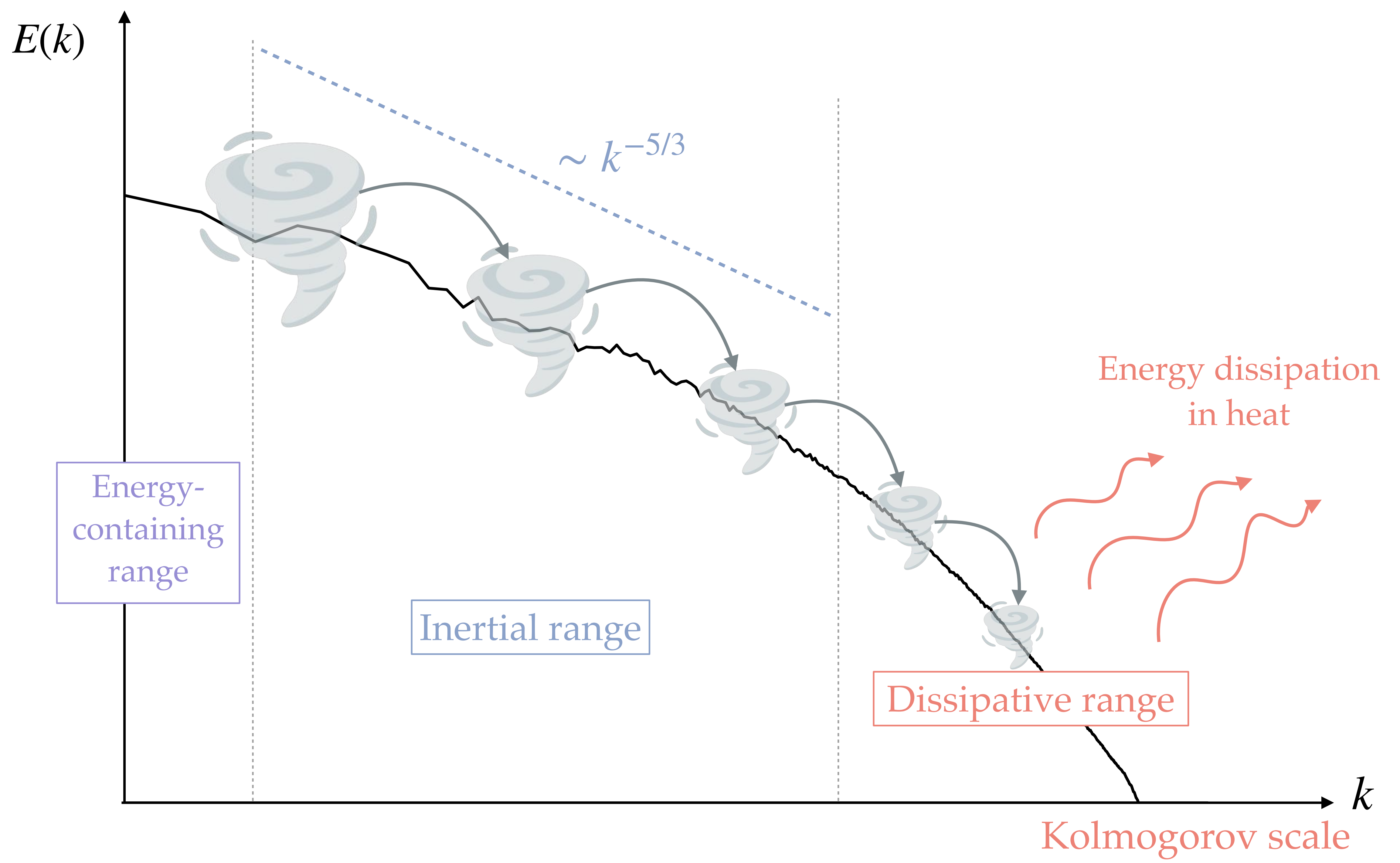
Context





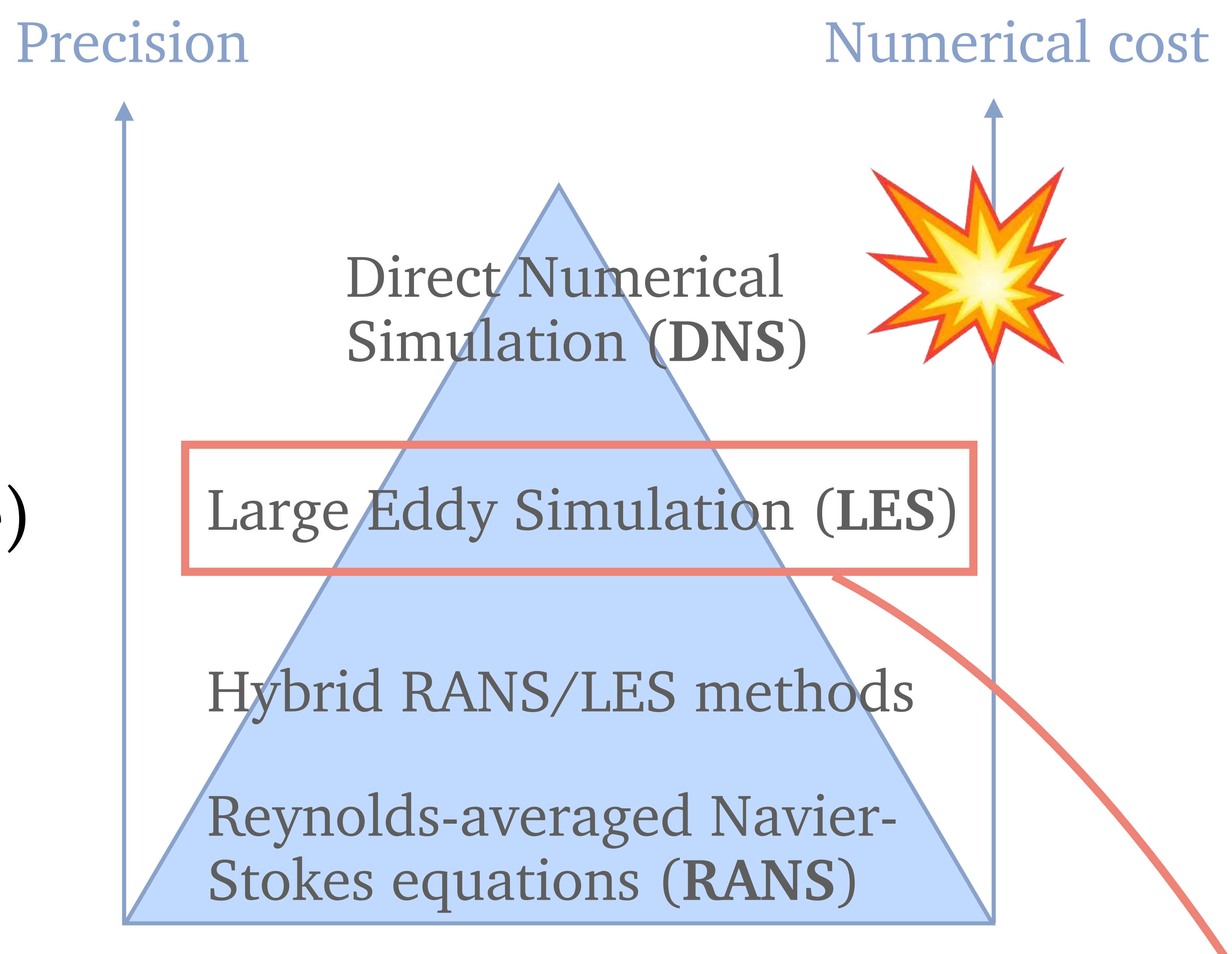


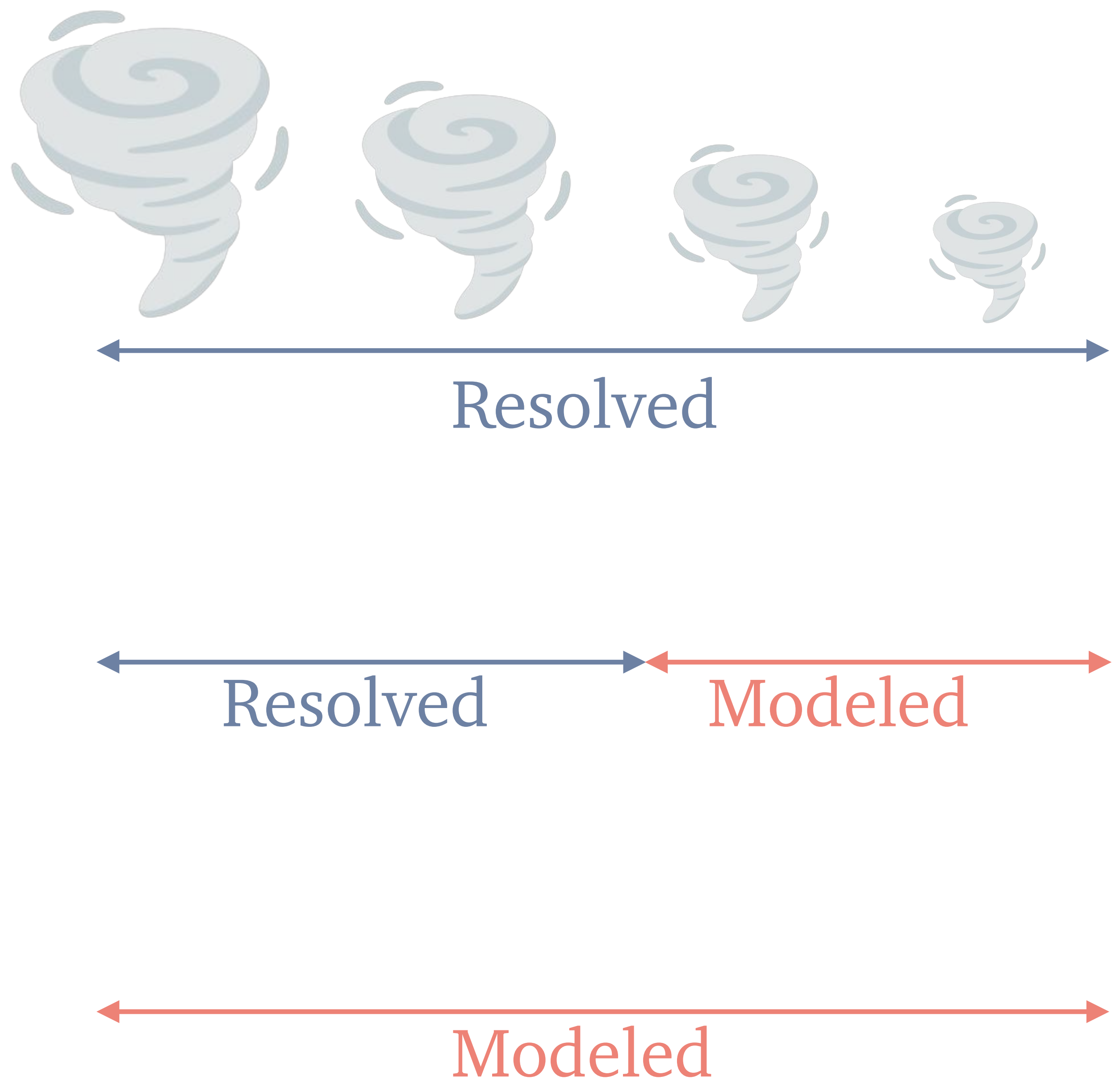
"Big whirls have little whirls
that feed on their velocity,
And little whirls have lesser whirls
and so on to viscosity."
- F. Richardson

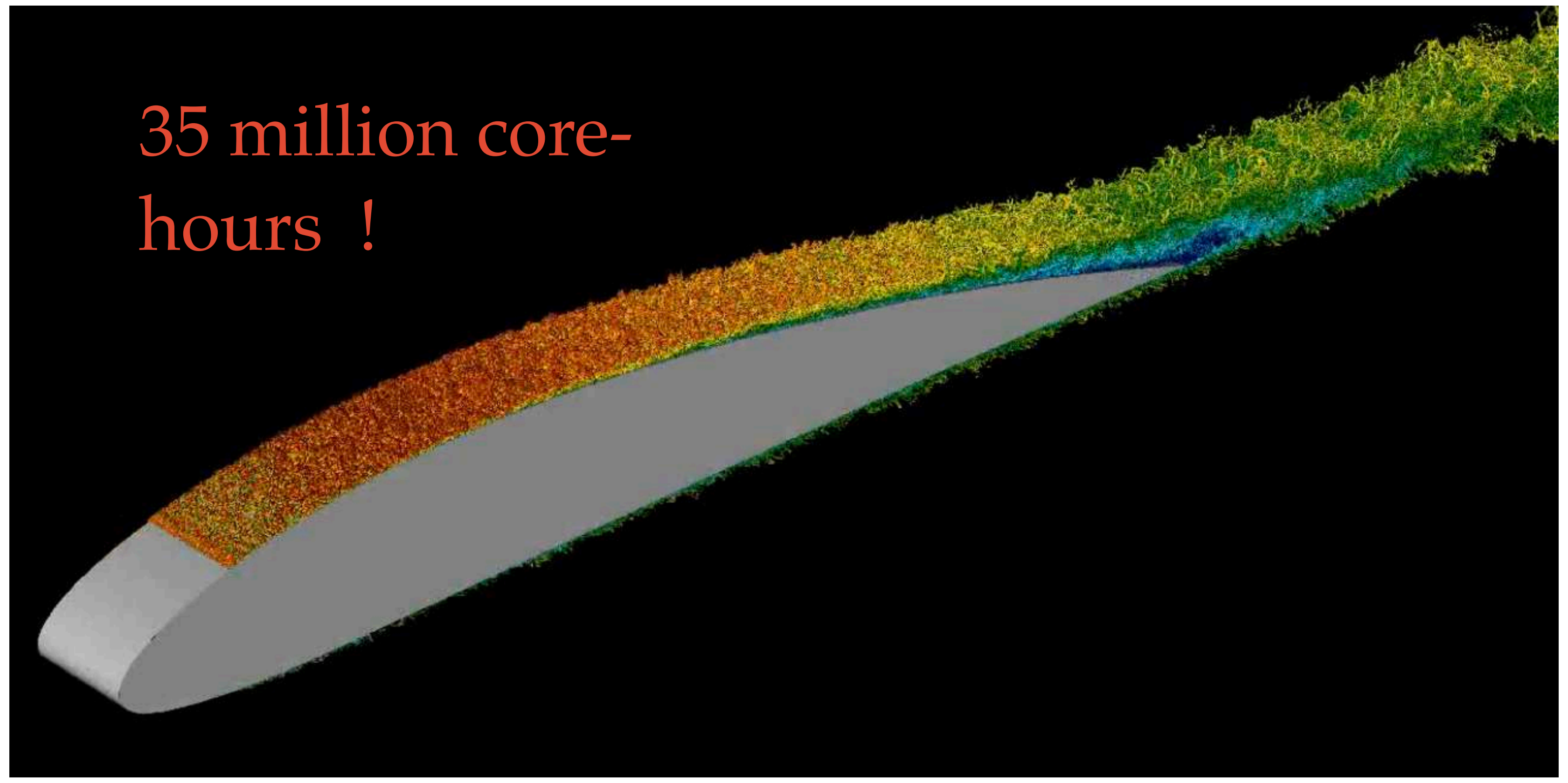
Context

Methods used to study turbulent flows :

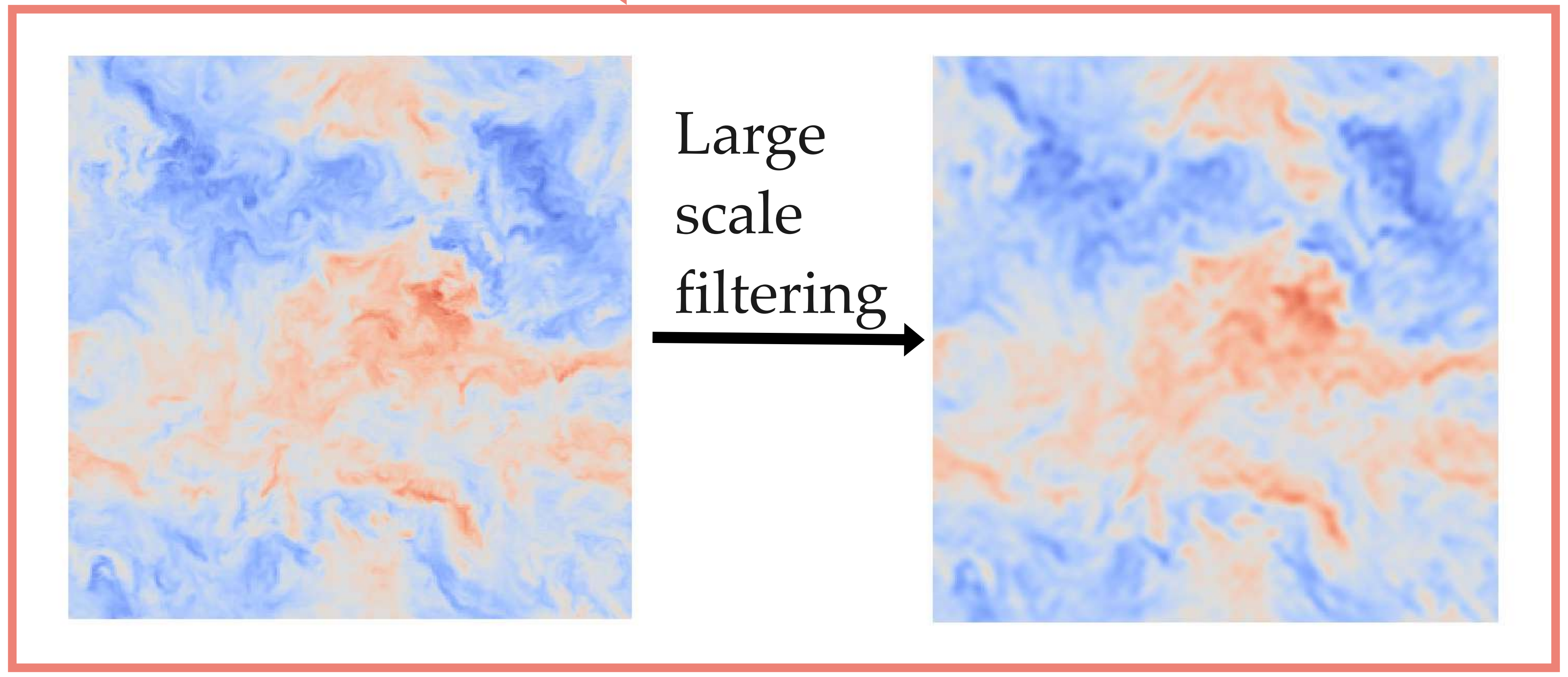
- Experimentation (costly / hard to reproduce)
- Numerical methods
    - DNS (costly)
    - RANS (most popular in industry)
    - LES (becoming more popular)







Hosseini et al. (2016)



Context

Why LES ?

- Good compromise between affordability and accuracy
- Increased interest as computational power grows

Current challenges in LES :

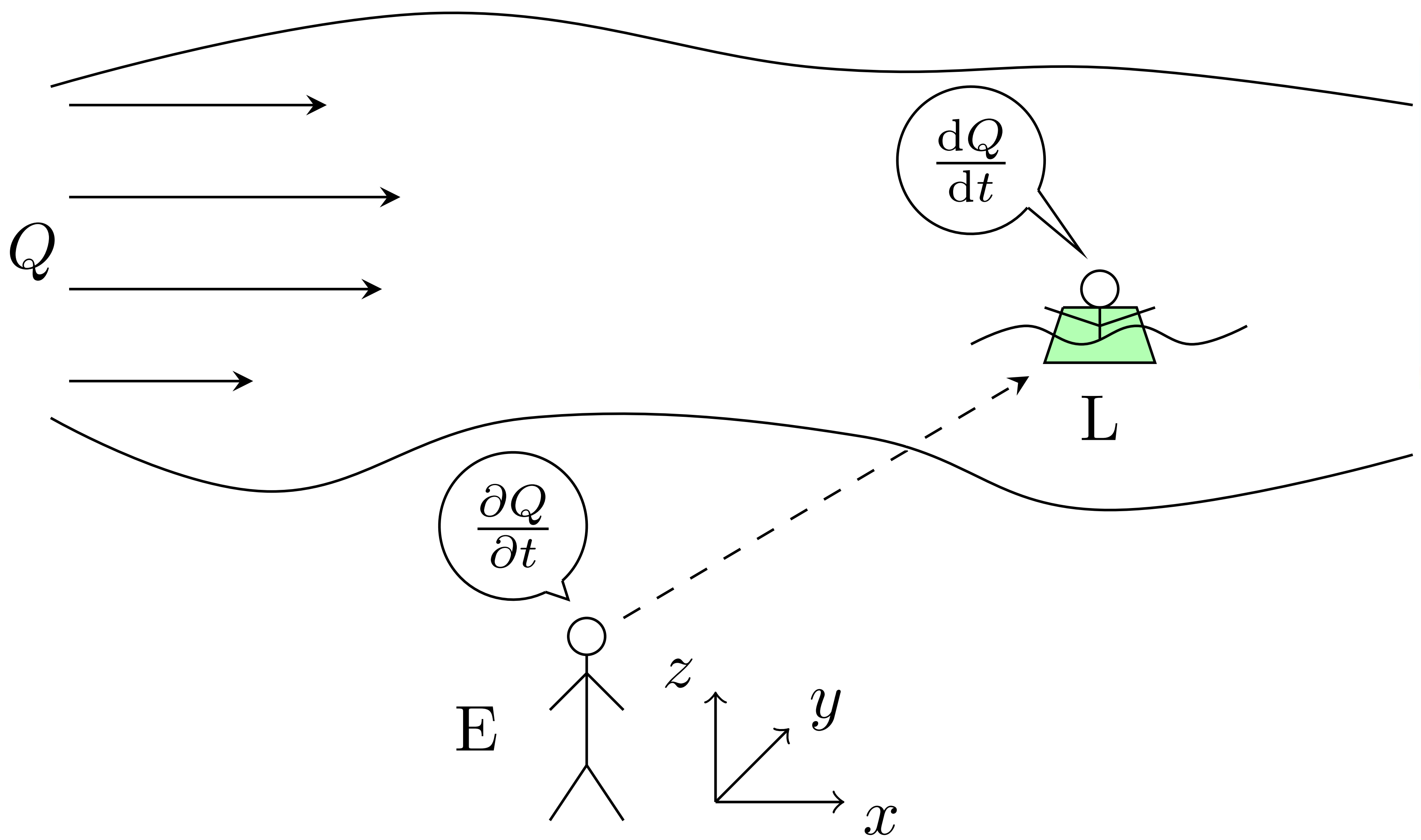
- Low dissipation, robust, multi-physics methods
- Design of effective subgrid-scale models
- Code optimization (faster grid generation, use of GPUs)

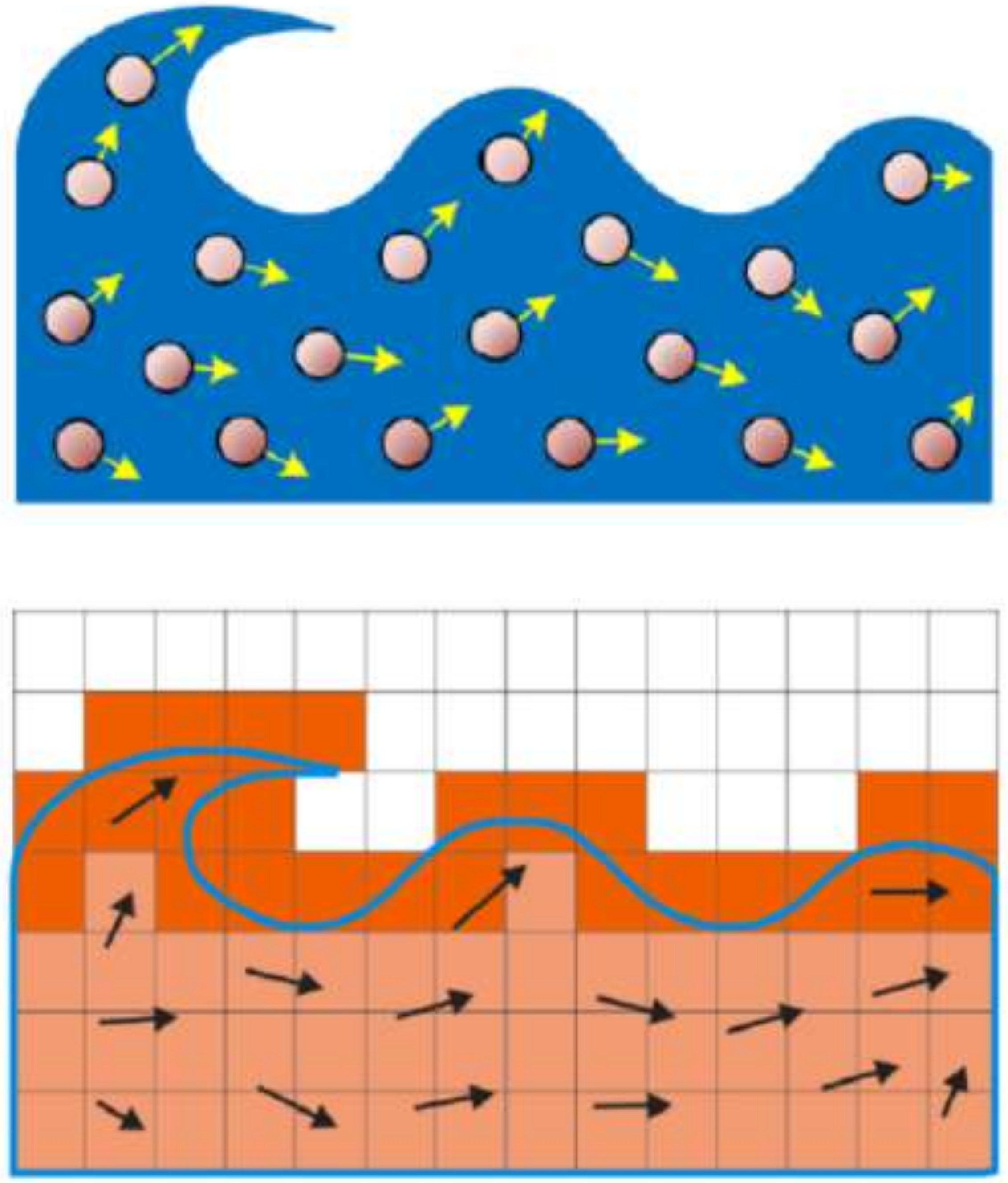
The performance and evaluation of SGS models cannot be separated from the numerical methods used to simulate the resolved scales

Context

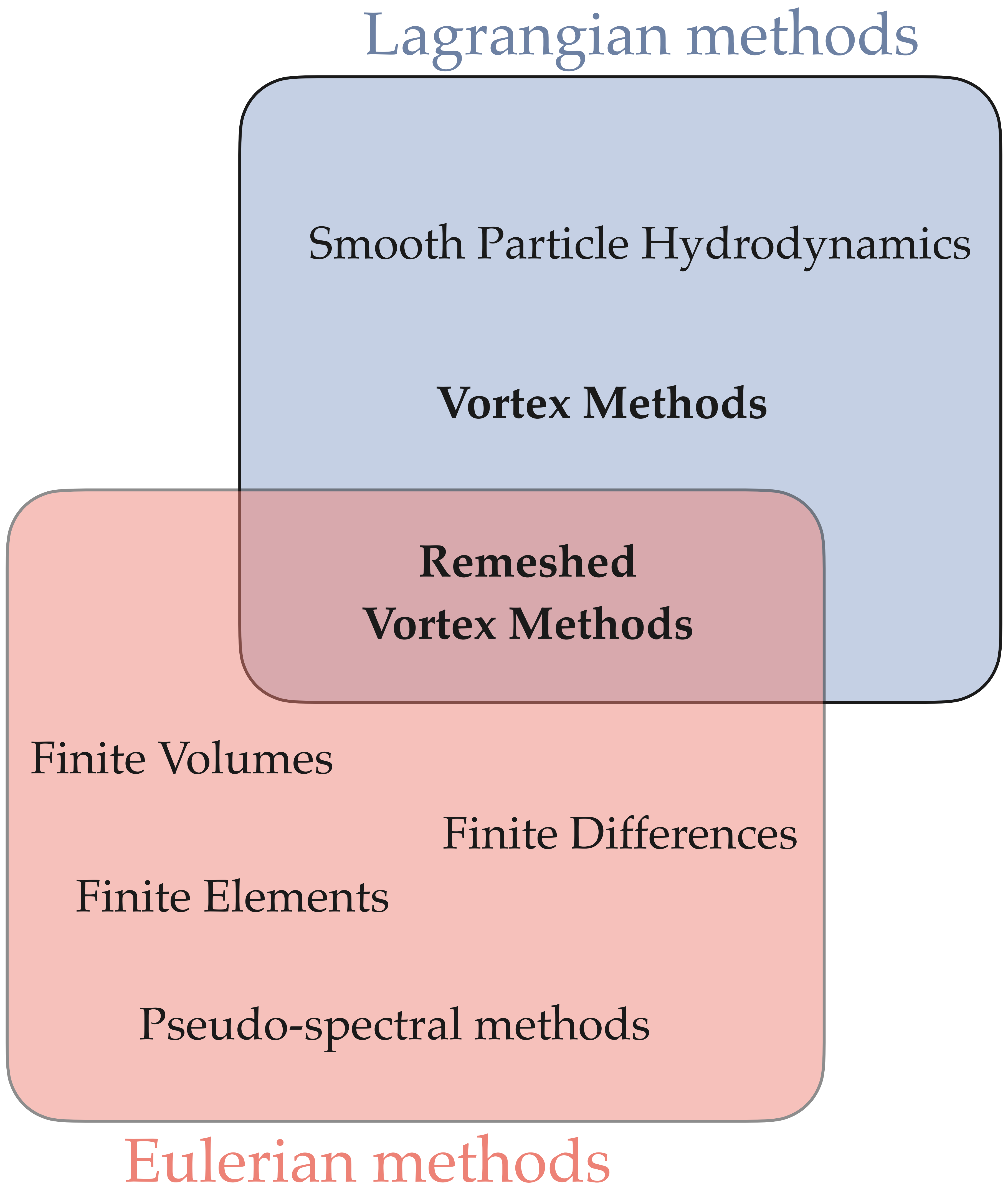
Numerical methods in Computational Fluid Dynamics can be sorted in two families, based on their frame of reference

- Lagrangian methods : particle-based
- Eulerian methods : grid-based





Liu et al. (2018)



Context

Methods

Remeshed Vortex Methods

Subgrid-Scale Modeling

Governing equations : incompressible Navier-Stokes equations

Vorticity (ω)-velocity(u) formulation :

$$\boldsymbol{\omega} := \nabla \times \mathbf{u}$$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \underbrace{\boxed{\mathbf{u} \cdot \nabla \boldsymbol{\omega}} - \boxed{\boldsymbol{\omega} \cdot \nabla \mathbf{u}}}_{\text{Non-linear terms}} - \boxed{\frac{1}{Re} \Delta \boldsymbol{\omega}} = 0$$

Vorticity advection / transport

Stretching (only in 3D)

Diffusion

Non-linear terms

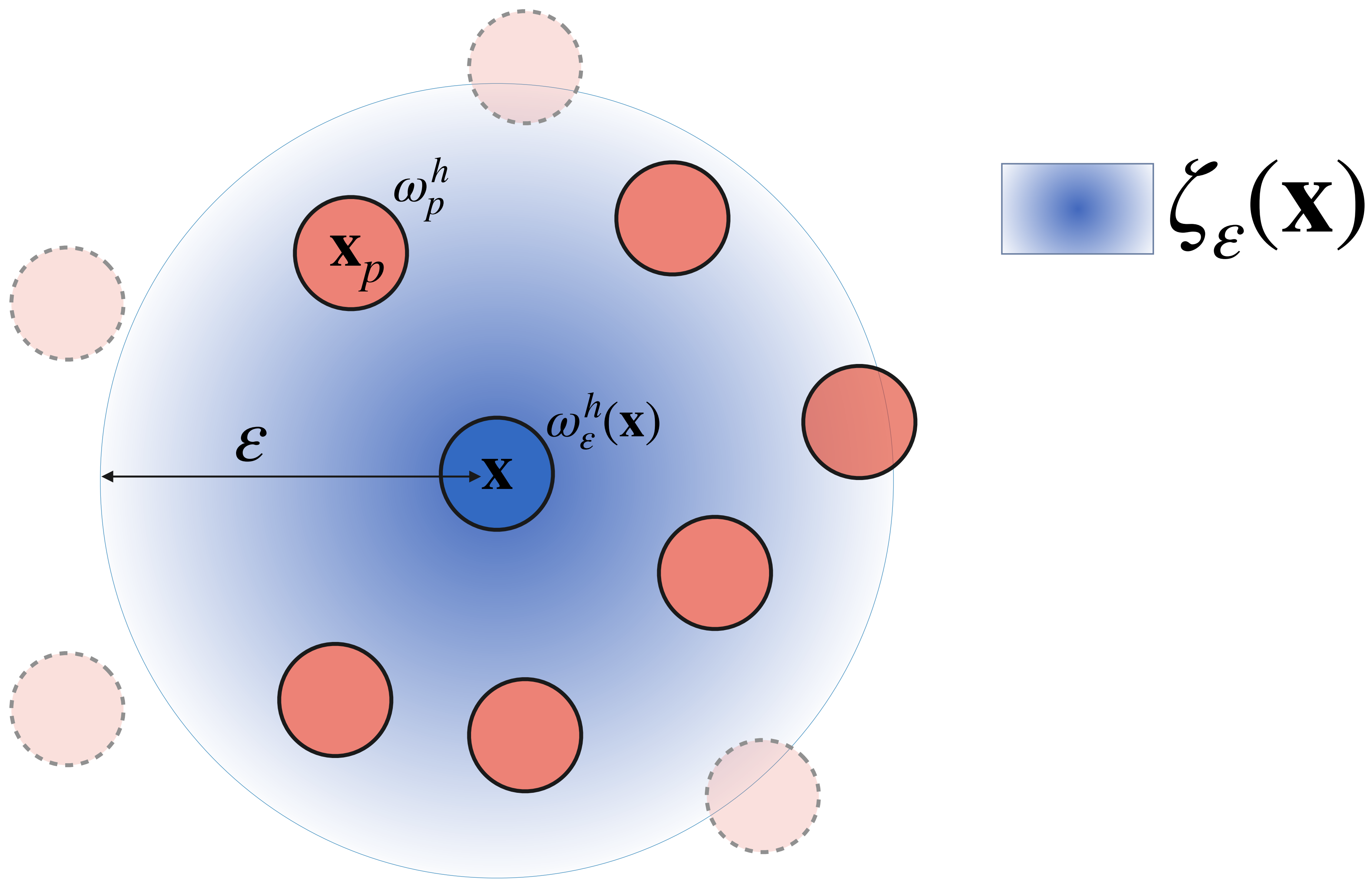
Velocity updated from vorticity through the **Poisson equation**

$$-\Delta \mathbf{u} = \nabla \times \boldsymbol{\omega}$$

(Deduced from the incompressibility condition)

+ Initial and boundary conditions

Lagrangian Vortex Methods



- Vorticity transport equation

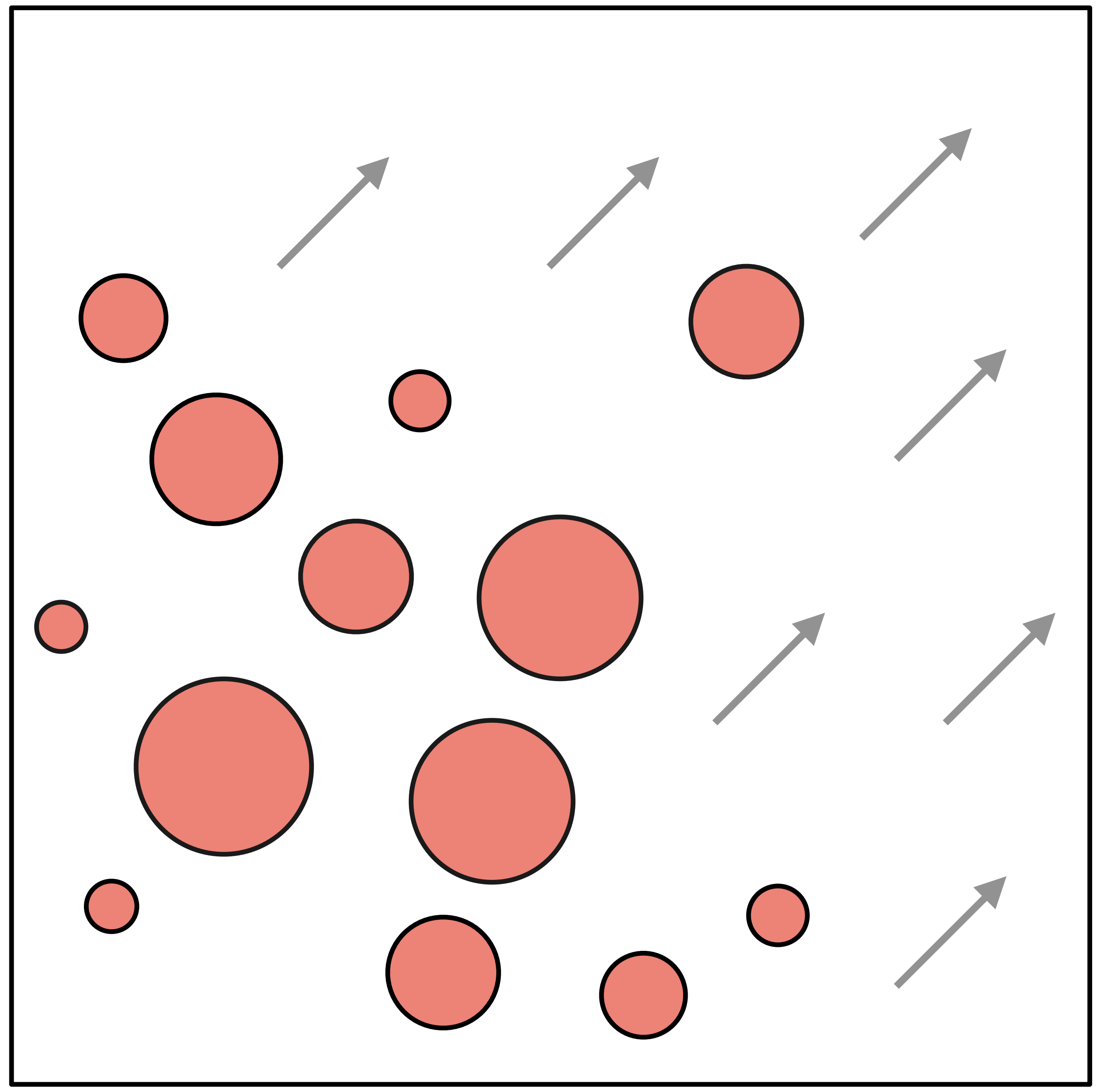
$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} = 0$$

- Vorticity discretized on particles

$$\boldsymbol{\omega}_\varepsilon^h(x,t) = \sum_{p \in P} \boldsymbol{\omega}_p^h(t) v_p \zeta_\varepsilon(\mathbf{x} - \mathbf{x}_p^h(t))$$

- Particles advected by velocity through

$$\begin{cases} \dfrac{d\mathbf{x}_p^h}{dt} = \mathbf{u}^h(\mathbf{x}_p^h(t), t) \\ \dfrac{d\boldsymbol{\omega}_p^h}{dt} = 0 \end{cases}$$



Stability condition

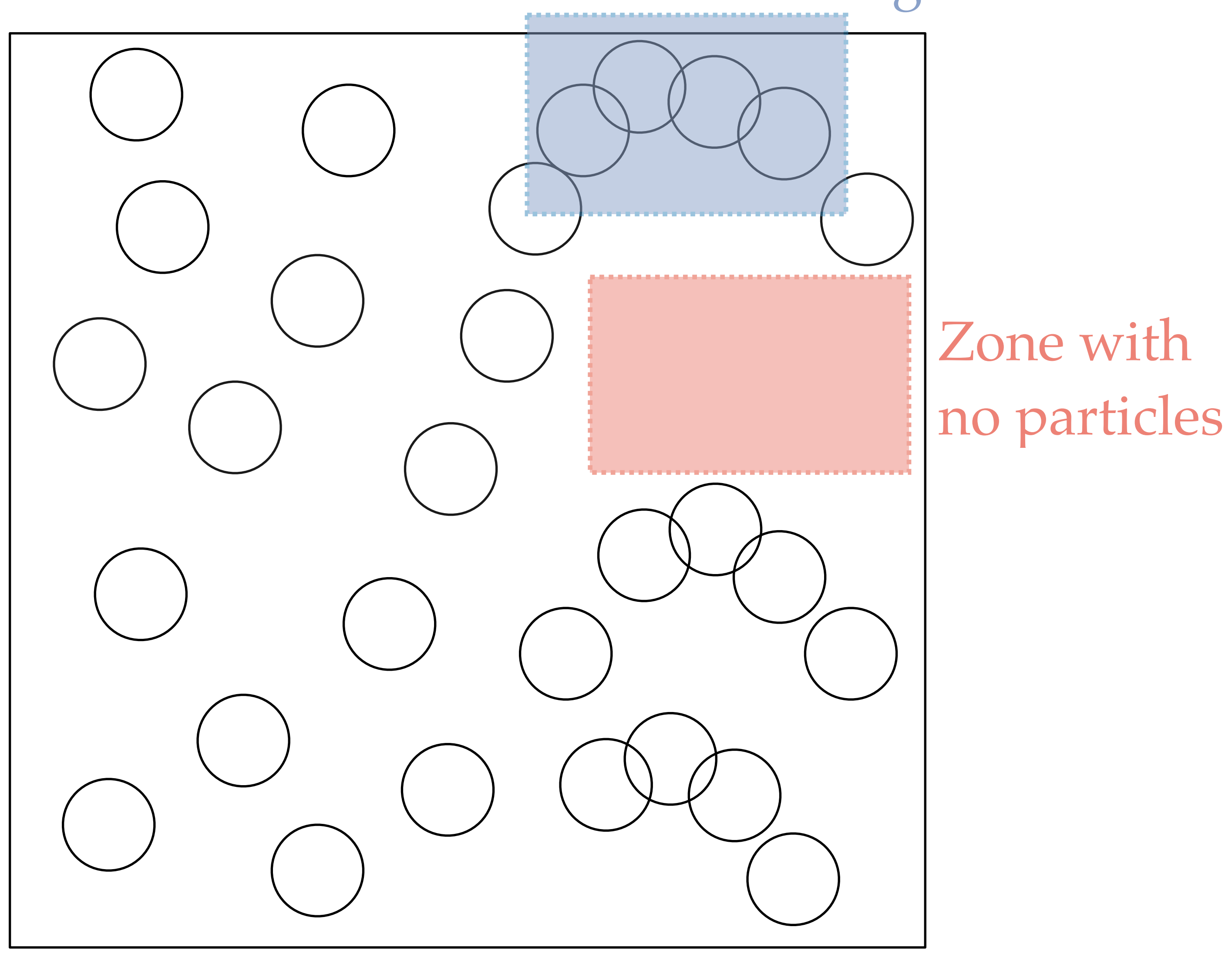
$$\Delta t = \frac{C_{LCFL}}{\|\nabla \mathbf{u}\|_\infty}$$

$$C_{LCFL} \leq 1$$

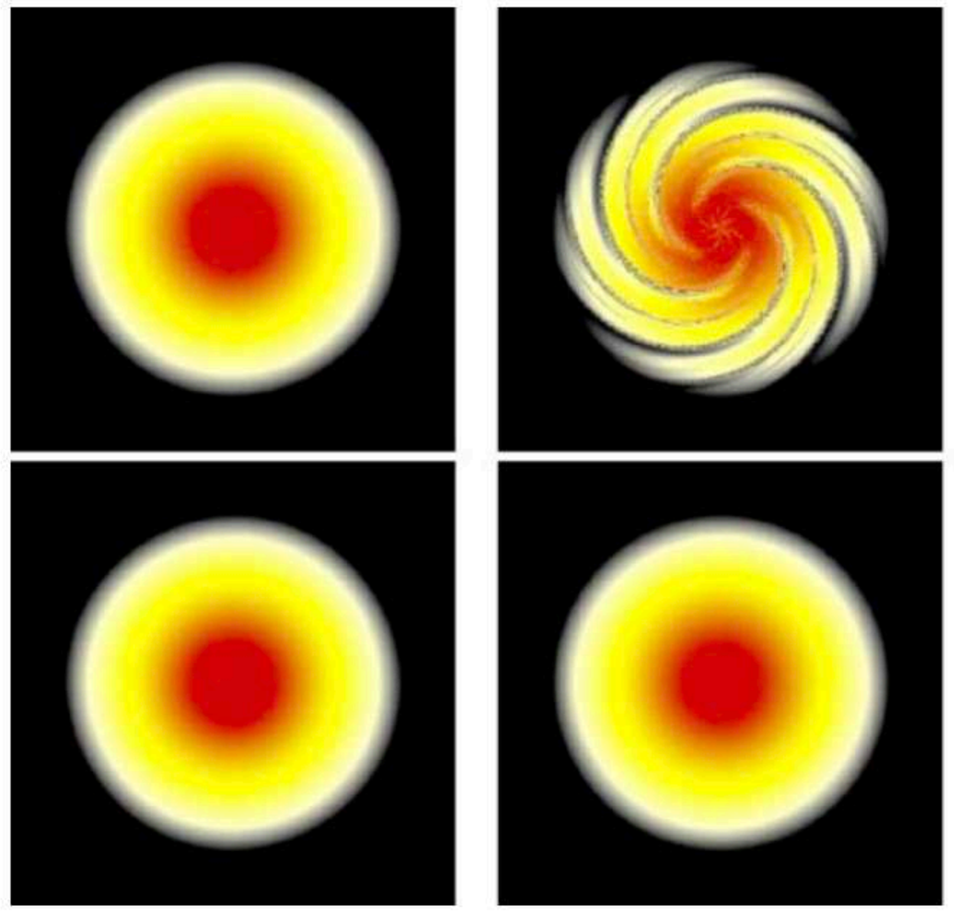
Chorin and Bernard (1973), Anderson and Greengard (1985)

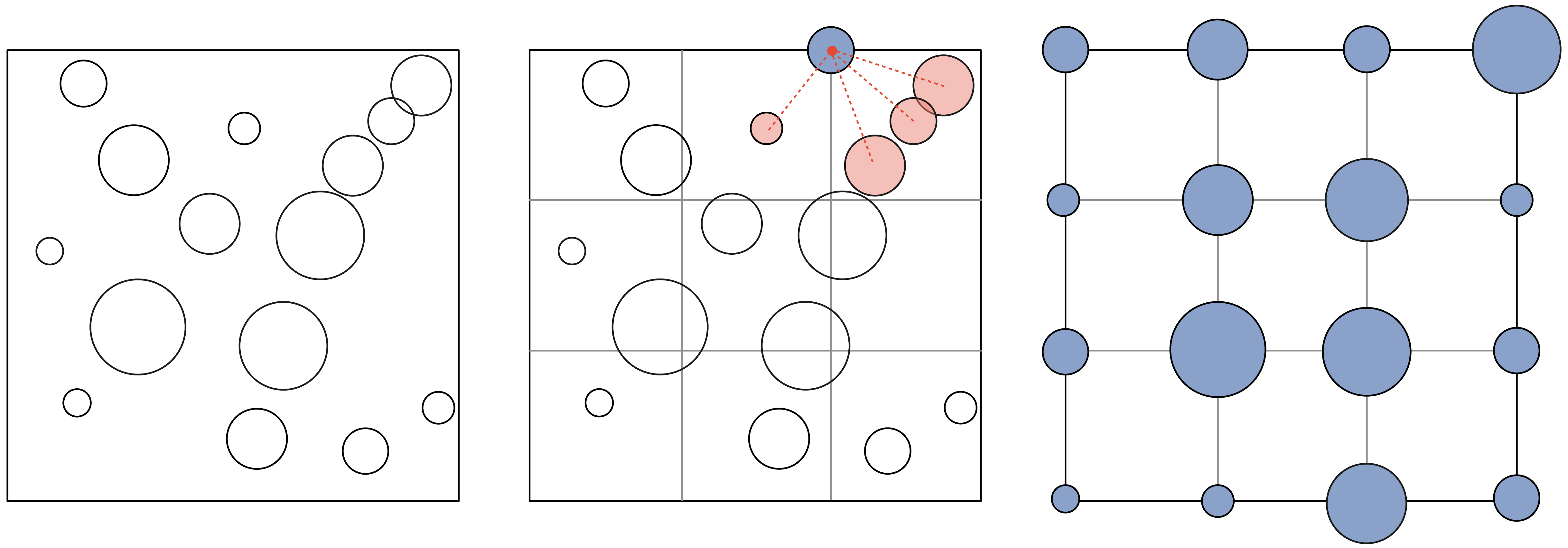
Remeshing

- Lagrangian Vortex Methods suffer from distortion effects
- The convergence of the method rely on being able to control the distance between particles
- Different techniques to correct this (particle strength correction…)
- The most popular consists in regridding (or **remeshing**) regularly the particles on an underlying Cartesian grid
  - The method is no longer grid-free
  - The remeshing function (or **remeshing kernel**) must be chosen with care



Rossinelli and Koumoutsakos (2008)

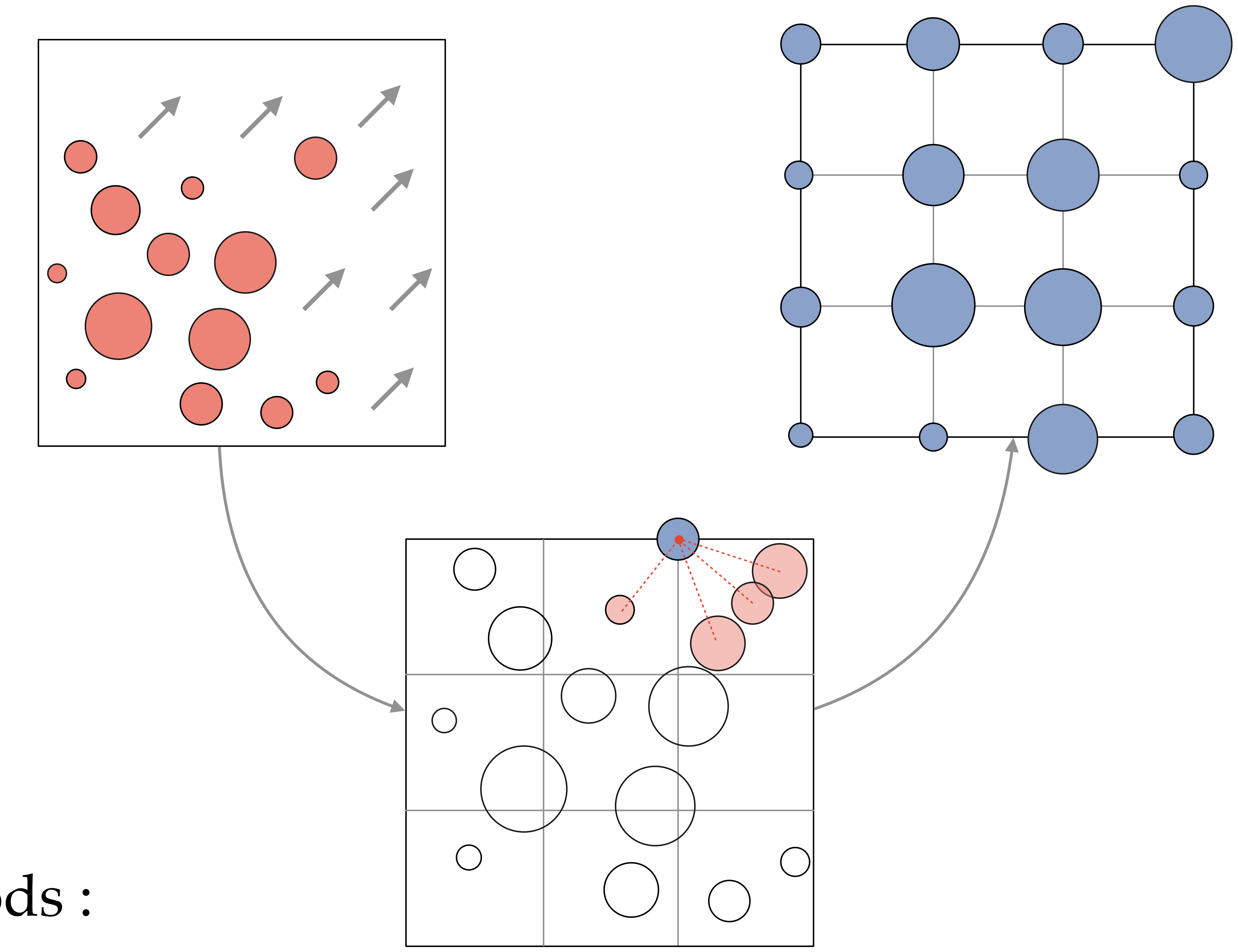




Huberson and Jolles (1990), Koumoutsakos and Leonard (1995)

Hybrid Vortex Methods

$$\begin{cases} \dfrac{\partial \boldsymbol{\omega}}{\partial t} + \boxed{\mathbf{u} \cdot \nabla \boldsymbol{\omega}} - \boxed{\boldsymbol{\omega} \cdot \nabla \mathbf{u}} - \boxed{\dfrac{1}{Re} \Delta \boldsymbol{\omega}} = 0 \\ \boxed{-\Delta \mathbf{u} = \nabla \times \boldsymbol{\omega}} \end{cases}$$



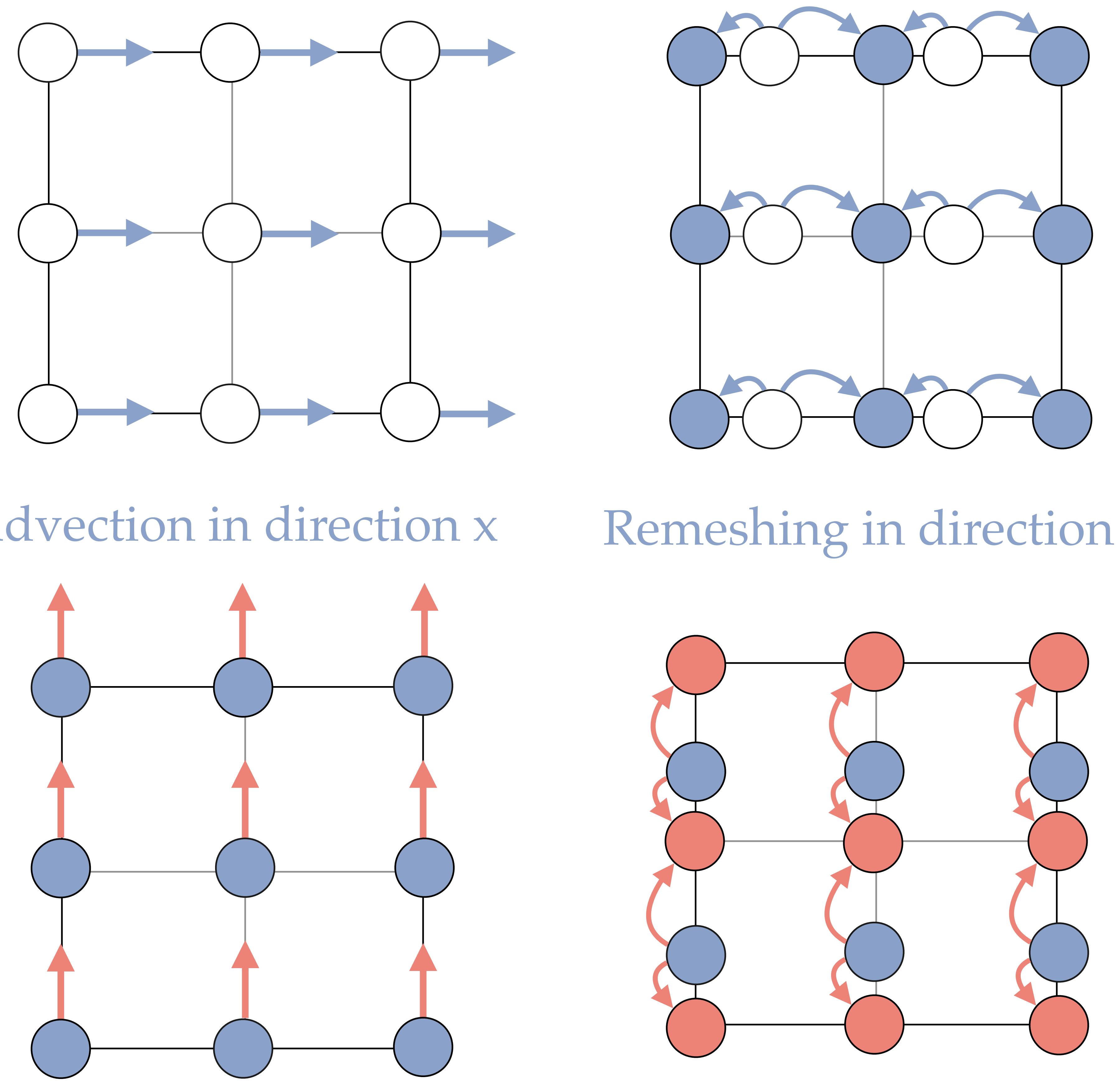
- Hybrid time-splitting scheme :
  - Lagrangian vorticity advection
  - Eulerian methods for the other steps
- Both frameworks are linked by the **remeshing** step
- Avoids common issues with classical Vortex methods : distortion effects, Poisson equation resolution…

Hysop

$$\begin{cases} \dfrac{\partial \boldsymbol{\omega}}{\partial t} + \boxed{\mathbf{u} \cdot \nabla \boldsymbol{\omega}} - \boxed{\boldsymbol{\omega} \cdot \nabla \mathbf{u}} - \boxed{\dfrac{1}{Re} \Delta \boldsymbol{\omega}} = 0 \\ \boxed{-\Delta \mathbf{u} = \nabla \times \boldsymbol{\omega}} \end{cases}$$

- Hybrid time-splitting scheme :
  - Lagrangian vorticity advection
  - Eulerian methods (finite differences and Fourier methods) for the other steps

- Directional Strang splitting for advection-remeshing and stretching operators



Advection in direction x

Remeshing in direction x

Advection in direction y

Remeshing in direction y

Hysop

initialization $\omega^0, \Delta t^0, t^0$

while $t^n < t^{end}$ do

Poisson — FFTW

Diffusion — FFTW, impl. Euler

for i in 0, d−1 do

Directional Advection_{i+1} — RK2

Directional Remeshing_{i+1} — $\Lambda_{4,2}$

end for

Stretching — 4th order FD, RK3

Adaptive time step — 4th order FD

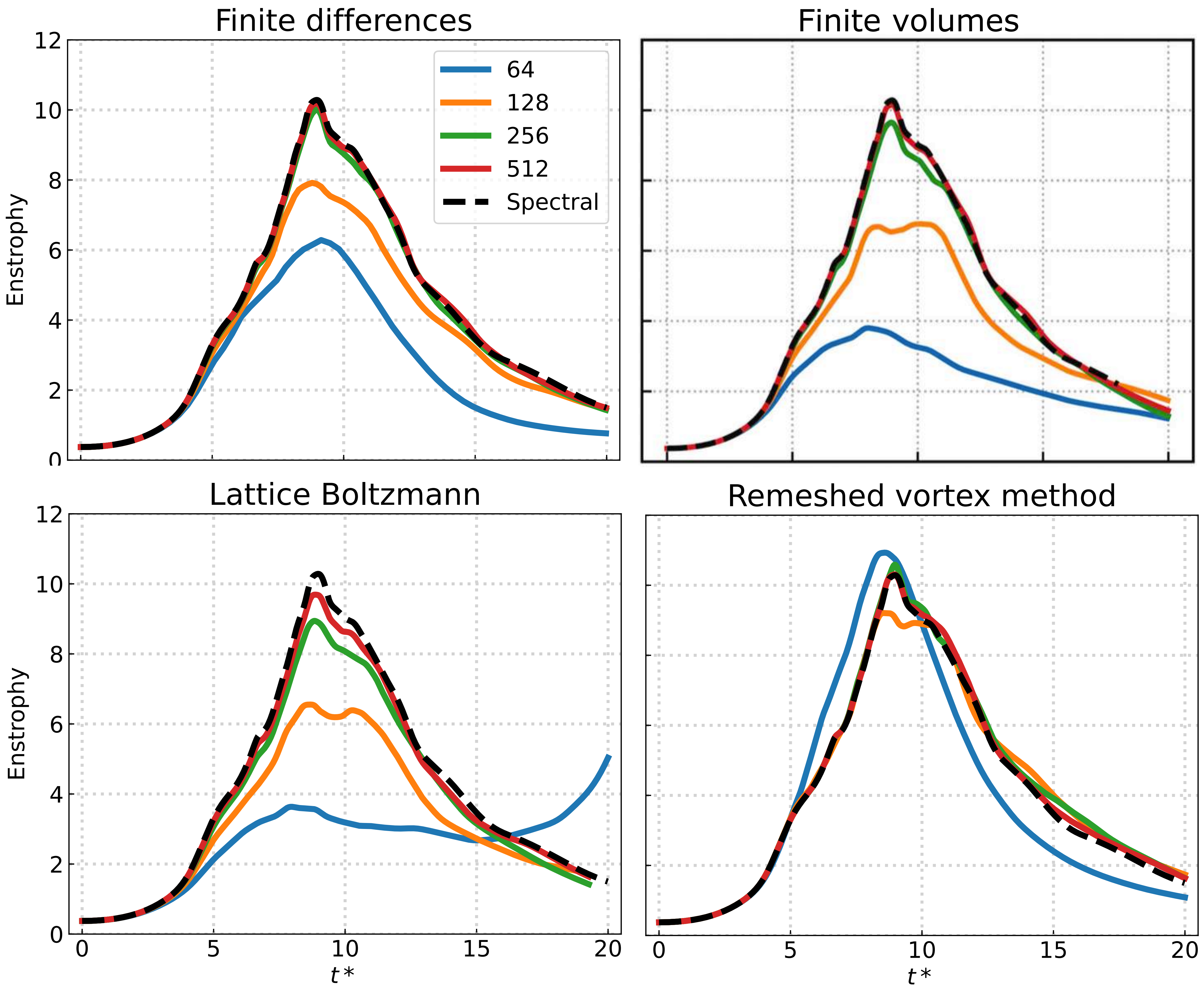
Why use RVMs?

Advantages

- Low dissipation
  due to the **Lagrangian treatment of advection** : good performances at low resolution compared to Eulerian methods
- Time step not tied to grid size but instead to velocity gradient
- Vorticity as a main variable : adapted to turbulent flows, no need for pressure boundary conditions
- Highly parallelizable

Limitations

- Restrained to Cartesian grids, challenging for complex geometries
- Between first and second order convergence in practise



TGV at Re=1600 : data from Jammy et al. *(2016)*, Suss et al. *(2023)* and Mimeau et al. *(2021)*

Remeshed Vortex Methods

- A priori advantageous for LES : low dissipation, vorticity as main variable
- Eulerian framework allows to easily integrate classical LES models
- Few works on LES with RVM

Goals of the thesis

- Assessment of the capacity of RVM to simulate turbulent flows in different configurations using LES
- Comparison and calibration of subgrid-scale models
- Sensitivity analysis of the LES to the model coefficients

Context

Methods

Remeshed Vortex Methods

Subgrid-Scale Modeling

Filtered Navier-Stokes equations

$$\begin{cases} \dfrac{\partial \overline{\boldsymbol{\omega}}}{\partial t} + \overline{\mathbf{u}} \cdot \nabla \overline{\boldsymbol{\omega}} - \overline{\boldsymbol{\omega}} \cdot \nabla \overline{\mathbf{u}} - \dfrac{1}{Re} \Delta \overline{\boldsymbol{\omega}} = \boxed{- \nabla \cdot R_{SGS}} \\ -\Delta \overline{\mathbf{u}} = \nabla \times \overline{\boldsymbol{\omega}} \end{cases}$$

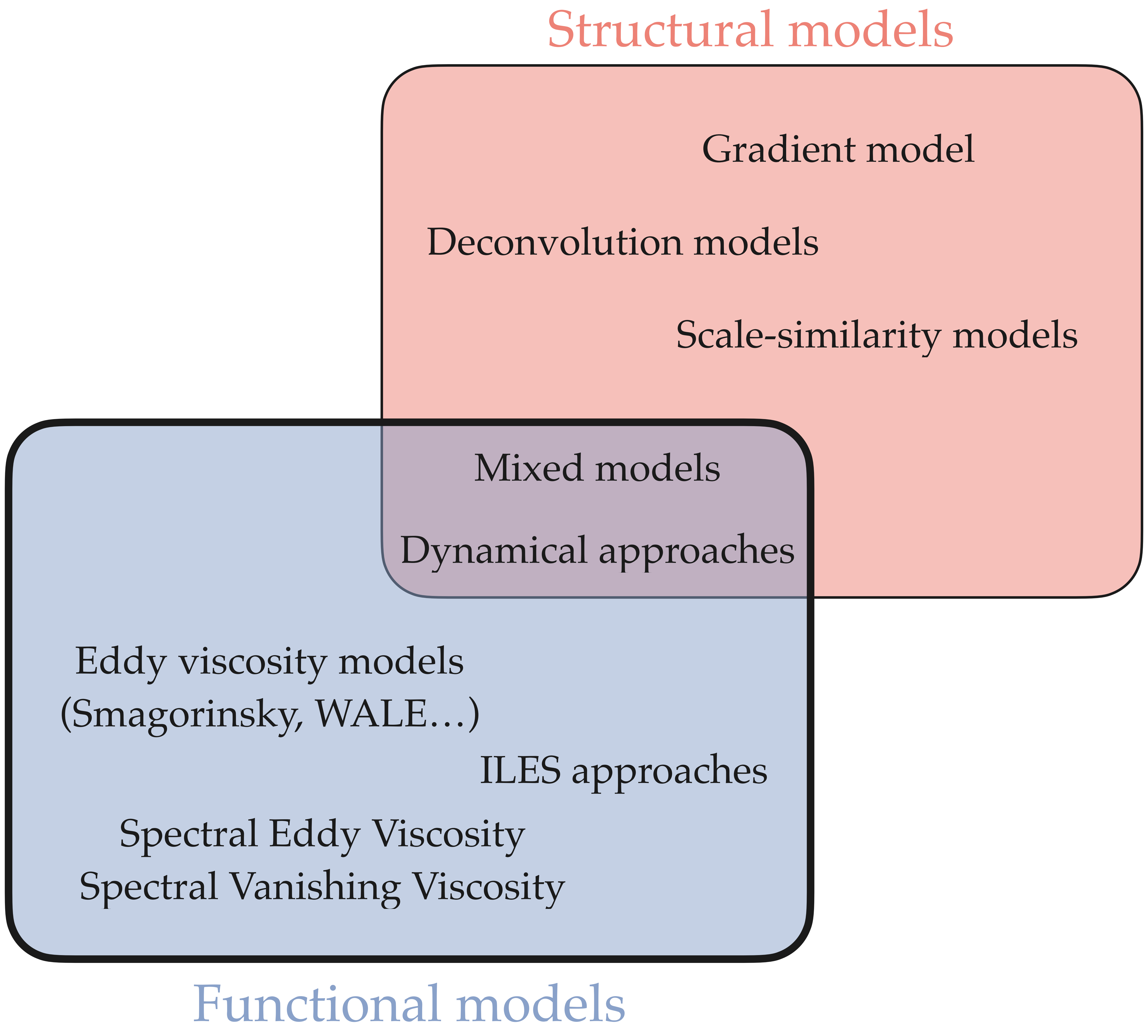
- $\overline{\boldsymbol{\omega}}$: filtered vorticity field
- $\overline{\mathbf{u}}$: filtered velocity field
- LES filter : implicit (grid filter)

Term requiring modelling

$$R_{SGS} = \underbrace{\boxed{\overline{\boldsymbol{\omega} \otimes \mathbf{u}} - \overline{\boldsymbol{\omega}} \otimes \overline{\mathbf{u}}}}_{\text{Advection}} - \underbrace{\boxed{\overline{\mathbf{u} \otimes \boldsymbol{\omega}} + \overline{\mathbf{u}} \otimes \overline{\boldsymbol{\omega}}}}_{\text{Stretching}}$$

LES models

- Structural models aim to directly model the subgrid-scale tensor by making assumptions about the LES filter
  - Often not diffusive enough and require some kind of regularization
- Functional models aim to model the diffusive effects of the missing small scales through an eddy viscosity operator (which can be implicit)
  - Most popular family of models



Eddy viscosity models

Eddy-viscosity model for velocity-pressure formulation

$$\tau_{SGS} = -\nu_{SGS}(\nabla\bar{\mathbf{u}} + \nabla\bar{\mathbf{u}}^T)$$

Eddy-viscosity model for vorticity-velocity formulation

$$\nabla \cdot R_{SGS} = \nabla \times \nabla \cdot \tau_{SGS}$$

$$\approx \nabla \times (\nu_{SGS}\nabla \times \bar{\boldsymbol{\omega}}) = -\nabla \cdot (\nu_{SGS}(\nabla\bar{\boldsymbol{\omega}} - \nabla\bar{\boldsymbol{\omega}}^T))$$

$$\Longrightarrow \boxed{R_{SGS} = -\nu_{SGS}\,(\nabla\bar{\boldsymbol{\omega}} - \nabla\bar{\boldsymbol{\omega}}^T)}$$

Smagorinsky (**Smag**)

$$\nu_{SGS} = (C_S\Delta)^2|S|$$

with $S = \frac{1}{2}(\nabla\bar{\mathbf{u}} + \nabla\bar{\mathbf{u}}^T)$

Note : model coefficient can be fixed or adapted dynamically

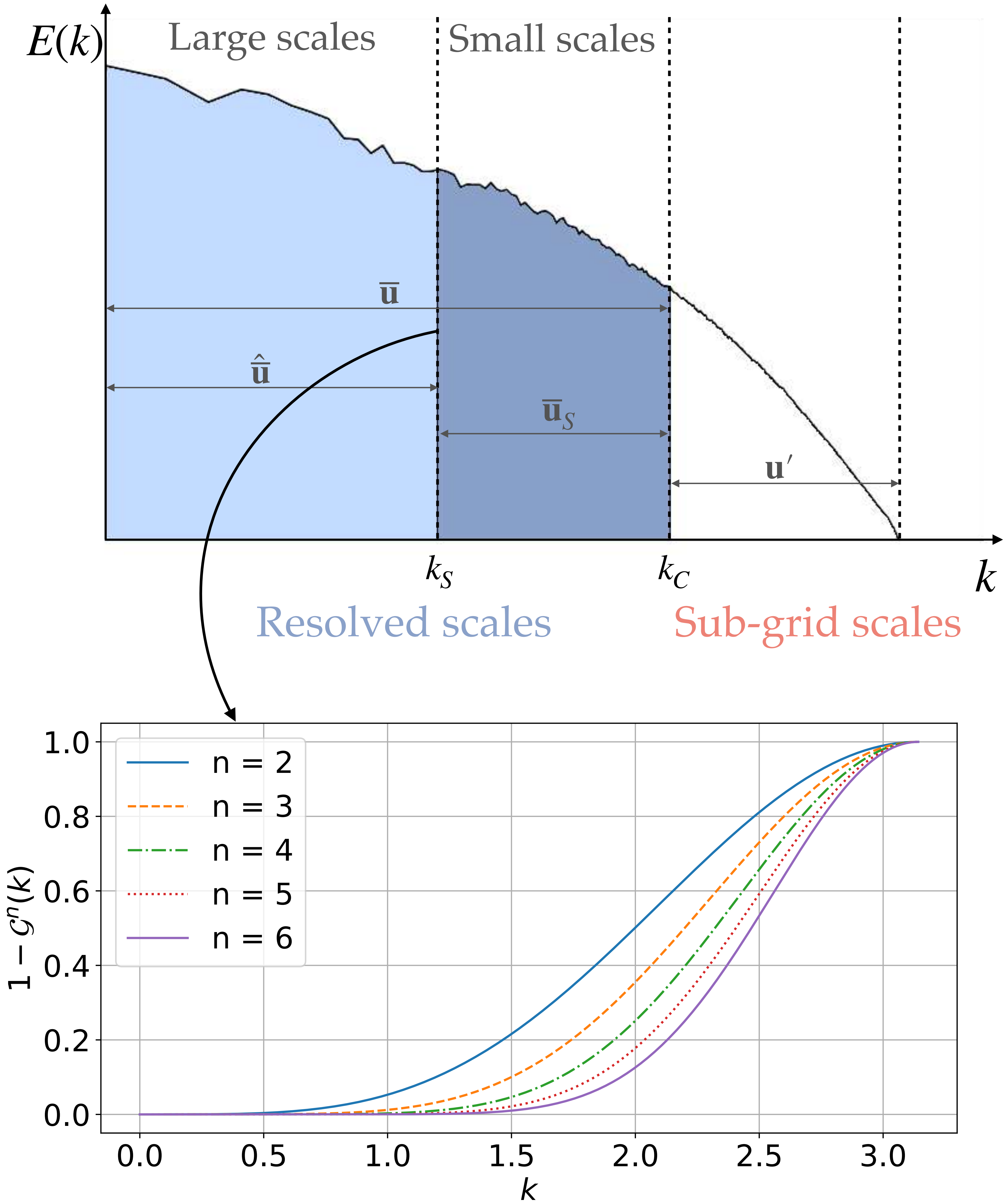
Mansour et al. (1978), Mansfield et al. (1998), Smagorinsky (1963)
Germano et al. (1991), Lilly (1992)

Variational multiscale models

$$R_{SGS} = -\nu_{SGS}(\nabla \overline{\boldsymbol{\omega}}_S - \nabla \overline{\boldsymbol{\omega}}_S^T)$$

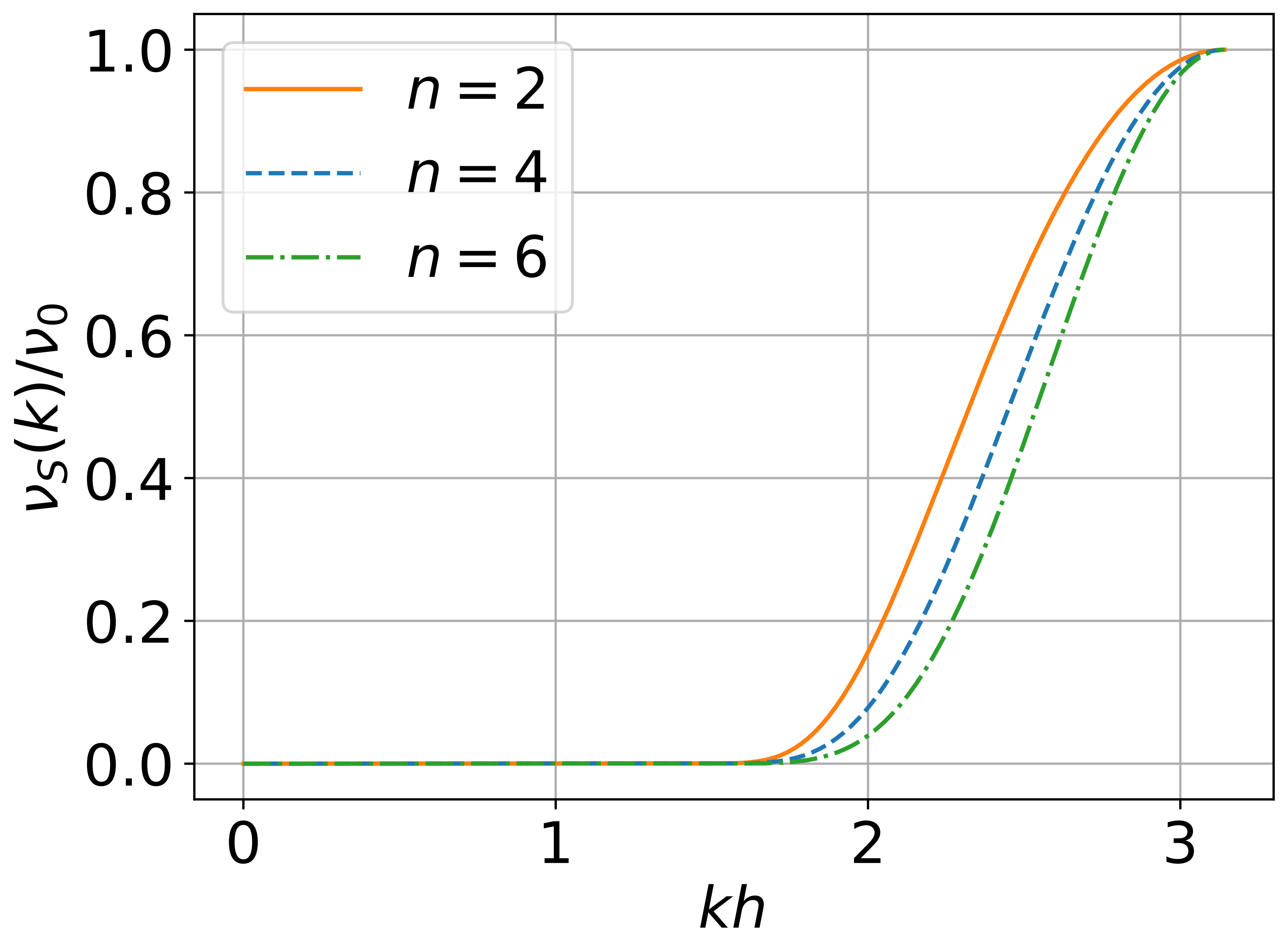
with $\overline{\boldsymbol{\omega}}_S$ the **small scales** of the **resolved** vorticity field

- Focuses the artificial viscosity to the end of the spectrum, leaving the large scales untouched → less dissipative than classical artificial viscosity models
- Requires an additional explicit filter



Hughes et al. (2001), Cocle et al. (2007)

Spectral vanishing viscosity



Modification of the spectral diffusion operator

$$\partial_t \hat{\boldsymbol{\omega}} = -\left(\frac{k_x^2}{Re} + \frac{k_y^2}{Re} + \frac{k_z^2}{Re}\right)\hat{\boldsymbol{\omega}}$$

to introduce viscosity in the small scales of the vorticity field

$$\partial_t \hat{\boldsymbol{\omega}} = -\left(k_x^2\left(\frac{1}{Re} + \nu_S(k_x, h_x)\right) + k_y^2\left(\frac{1}{Re} + \nu_S(k_y, h_y)\right) + k_z^2\left(\frac{1}{Re} + \nu_S(k_z, h_z)\right)\right)\hat{\boldsymbol{\omega}}$$

with

$$\nu_s(k, h) = \frac{C_{SVV}}{k_c} \times \exp\left[-\left(\frac{k_c - k}{0.3k_c - k}\right)^2\right](1 - \mathcal{G}^n(kh))$$

where $\mathcal{G}^n$ is the same low-pass filter of order n as for explicit filtering in VMS.

$\Longrightarrow$ **similar to VMS approach but at lower cost (integrated directly in the diffusion step)**

Tadmor (1989), Maday and Tadmor (1989), Karamanos and Karniadakis (2000), Kirby and Karniadakis (2002), Kirby and Sherwin (2006)

Hysop

initialization $\omega^0, \Delta t^0, t^0$

while $t^n < t^{end}$ do

- Poisson — FFTW
- Diffusion + SVV — FFTW, impl. Euler

for i in 0, d−1 do

- Directional Advection_{i+1} — RK2
- Directional Remeshing_{i+1} — $\Lambda_{4,2}$

end for

- Stretching — 4th order FD, RK3
- **Small-scale filtering** — FFTW
- **Eddy viscosity models** — 4th order FD, RK3
- Adaptive time step — 4th order FD

Context

Methods

Remeshed Vortex Methods

Subgrid-Scale Modeling

Homogeneous Isotropic Turbulence

Model benchmark

Model calibration

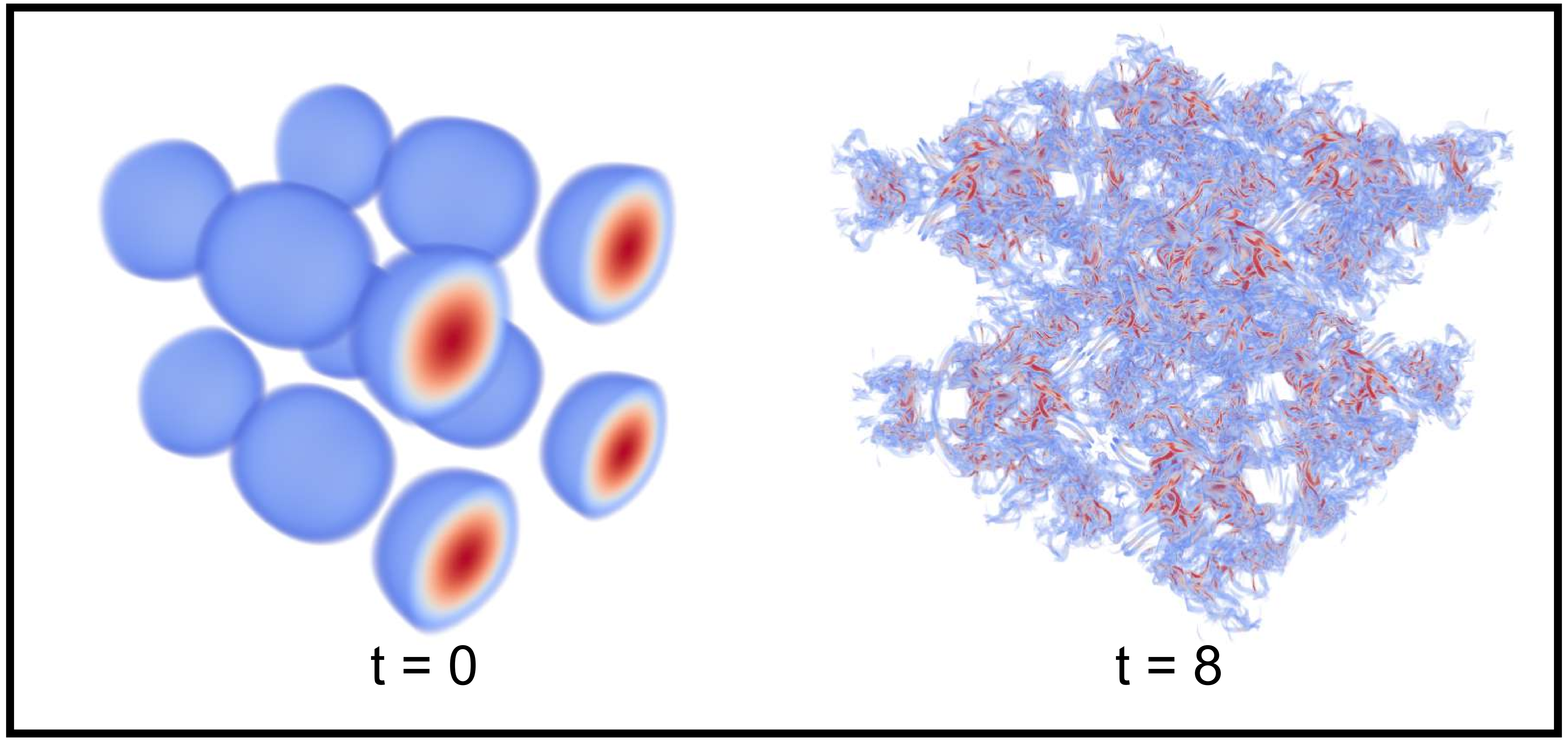
Wall-bounded flows

Flow over periodic hills

Flow past a sphere

Taylor-Green Vortex (TGV)

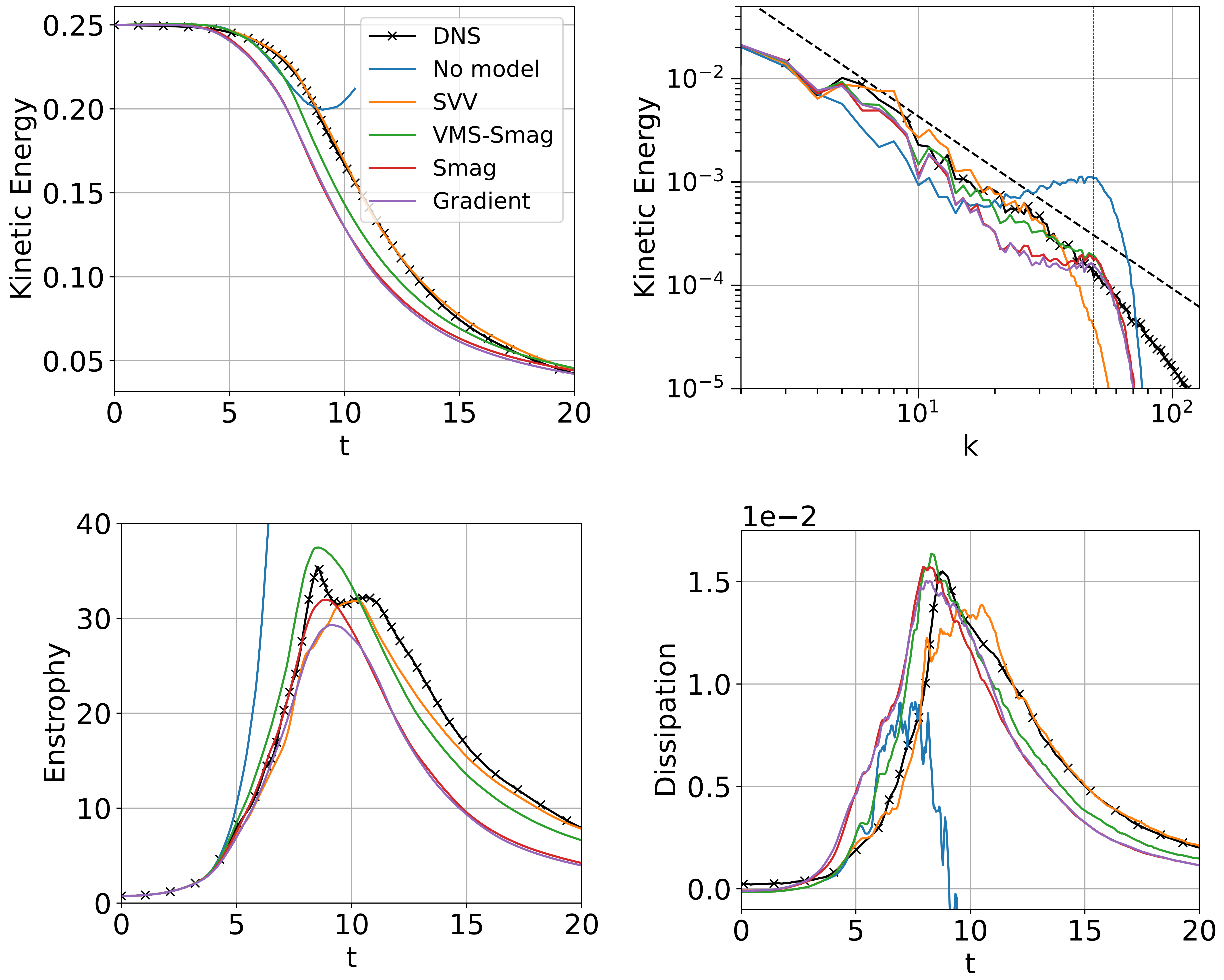
- Cubic domain
- Periodic boundary conditions
- Growth of turbulence from initial conditions
- Re=5000 and $N_{LES}^3 = 96^3$
- Need for a subgrid-scale model
- Comparison with present DNS and Dairay et al. (2014)



Results

- No model simulations are not stable
- Smag and Gradient model give similar results: too diffusive in the large resolved scales
- Models based on small-scale filtering (VMS-Smag and SVV) perform the best
- SVV is the most cost-effective as it does not require an additional operator

Model	Normalized av. time per iteration
No model	1
Smag	1.14
VMS-Smag	1.2
Gradient	1.19
SVV	1.02



VMS-Smag

$$R_{SGS} = -\nu_{SGS}(\nabla \overline{\boldsymbol{\omega}}_S - \nabla \overline{\boldsymbol{\omega}}_S^T)$$

$$\hat{\boldsymbol{\omega}}_S = (1 - \mathcal{G}^n(kh))\hat{\boldsymbol{\omega}}$$

$$\nu_{SGS} = (C_S \Delta)^2 |S|$$

Two parameters to calibrate for each model

SVV

$$\partial_t \hat{\boldsymbol{\omega}} = -(k_x^2(1/Re + \nu_S(k_x, h_x)) + k_y^2(1/Re + \nu_S(k_y, h_y)) + k_z^2(1/Re + \nu_S(k_z, h_z)))\hat{\boldsymbol{\omega}}$$

$$\nu_s(k, h) = \frac{C_{SVV}}{k_c} \times \exp\left[-\left(\frac{k_c - k}{0.3k_c - k}\right)^2\right](1 - \mathcal{G}^n(kh))$$

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Polynomial Chaos Expansion (PCE)

Uncertain parameters

$y_1, y_2, \ldots, y_n$

Quantity of interest

$q(y_1, y_2, \ldots, y_n)$

PCE : given polynomial basis $(\Psi_i)_i$ find coefficients $(\hat{q}_i)_i$ such that

$$q(y_1, \ldots, y_n) = \sum \hat{q}_i \Psi_i(y_1, \ldots, y_n)$$

Estimation of mean and variance of $q(y_1, y_2, \ldots, y_n)$

Stochastic calibration : if $q(y_1, y_2, \ldots, y_n)$ is the discrepancy with the reference value we can find coefficients minimizing it

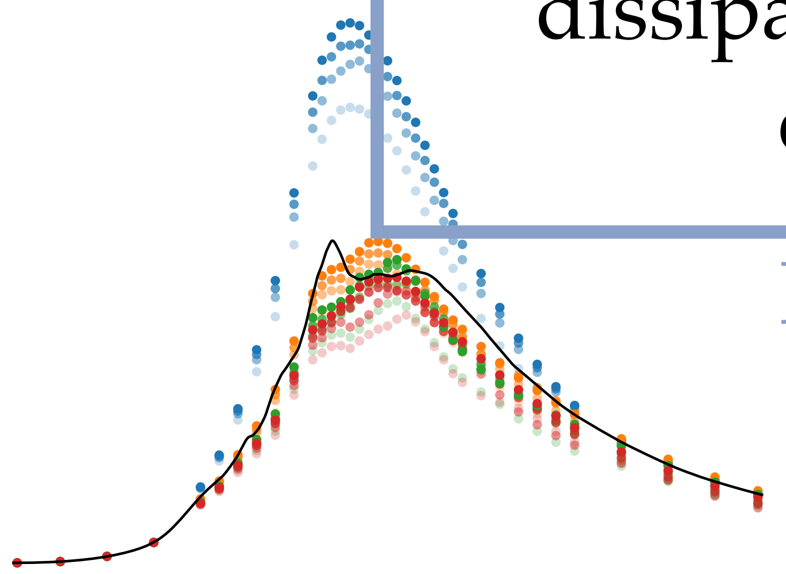
Xiu and Karnadakis (2002), Lucor et al. (2007), Xiao and Cinnella (2019)

- Focus on two models (VMS-Smag and SVV) that performed the best on Taylor-Green vortex
- Both models depend on two parameters: model coefficient and explicit filter order $\Longrightarrow$ Polynomial chaos expansion to calibrate the model parameters

Uncertain parameters

-model coefficients : C_{SVV} or C_S
-filter order : n

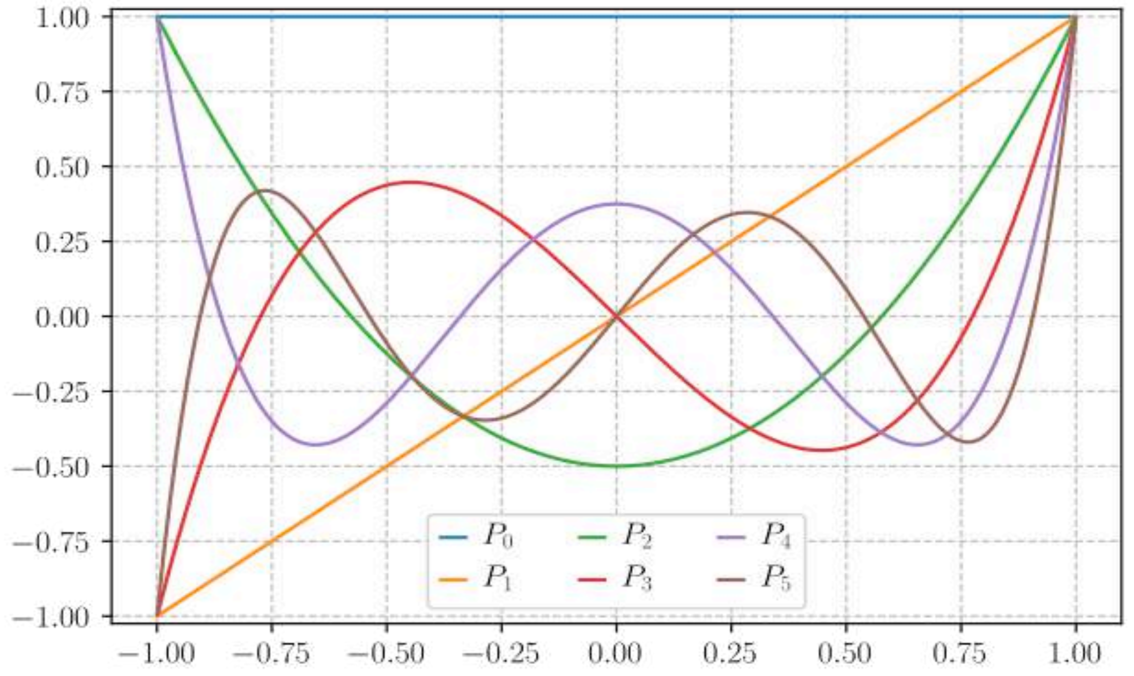
Quantity of interest $q(C, n)$ **obtained from LES** (e.g. kinetic energy dissipation and spectra, enstrophy)



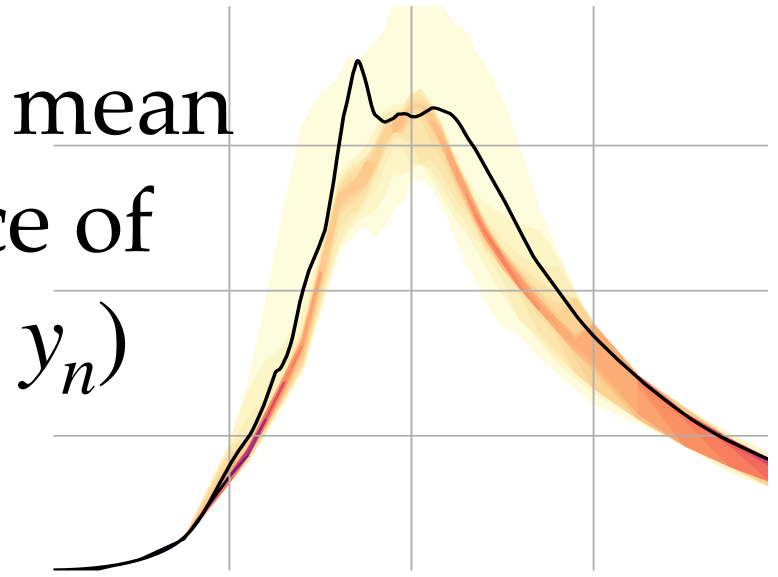
Hysop

Dakota open source software

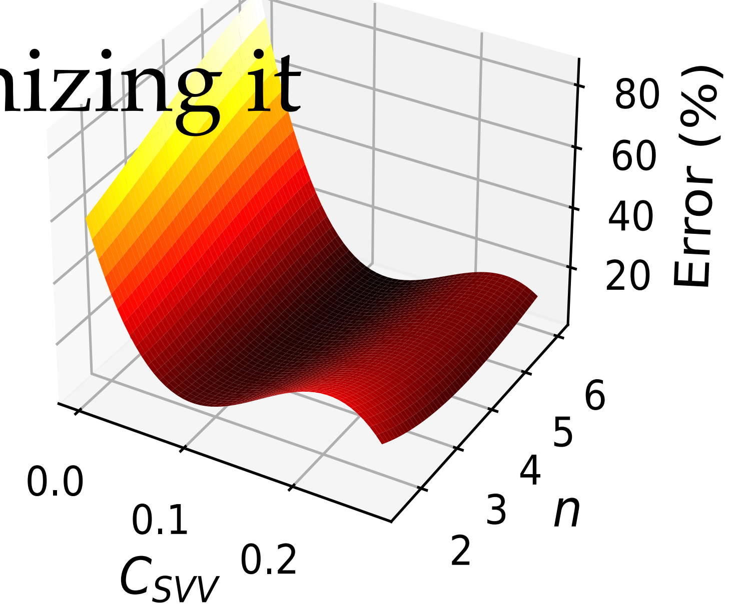
PCE : given polynomial basis $(\Psi_i)_i$ find coefficients $(\hat{q}_i)_i$ such that
$$q(y_1, \ldots, y_n) = \sum \hat{q}_i \Psi_i(y_1, \ldots, y_n)$$



Estimation of mean and variance of $q(y_1, y_2, \ldots, y_n)$

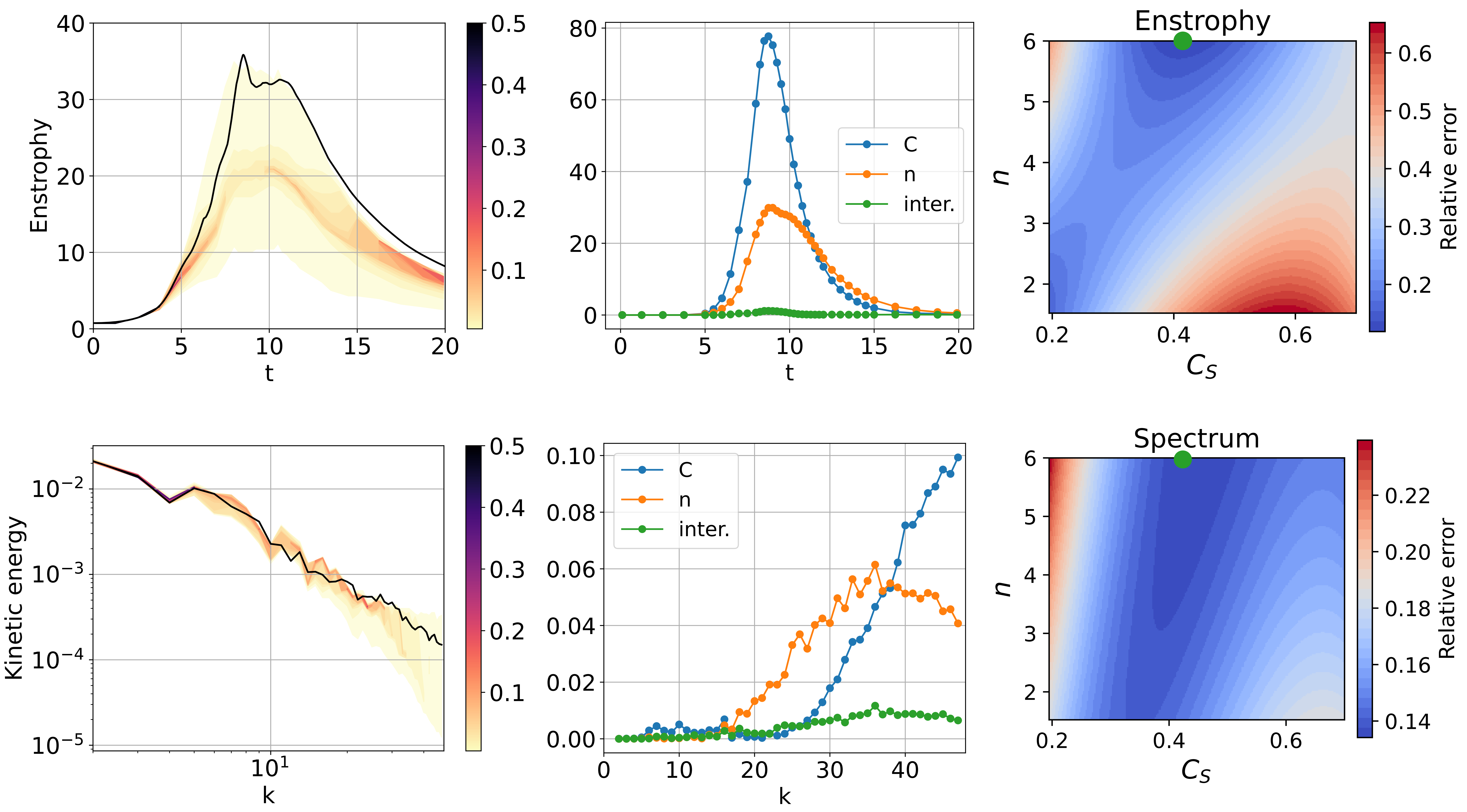


Stochastic calibration : if $q(y_1, y_2, \ldots, y_n)$ is the discrepancy with the reference value we can find coefficients minimizing it



Results for VMS-Smag

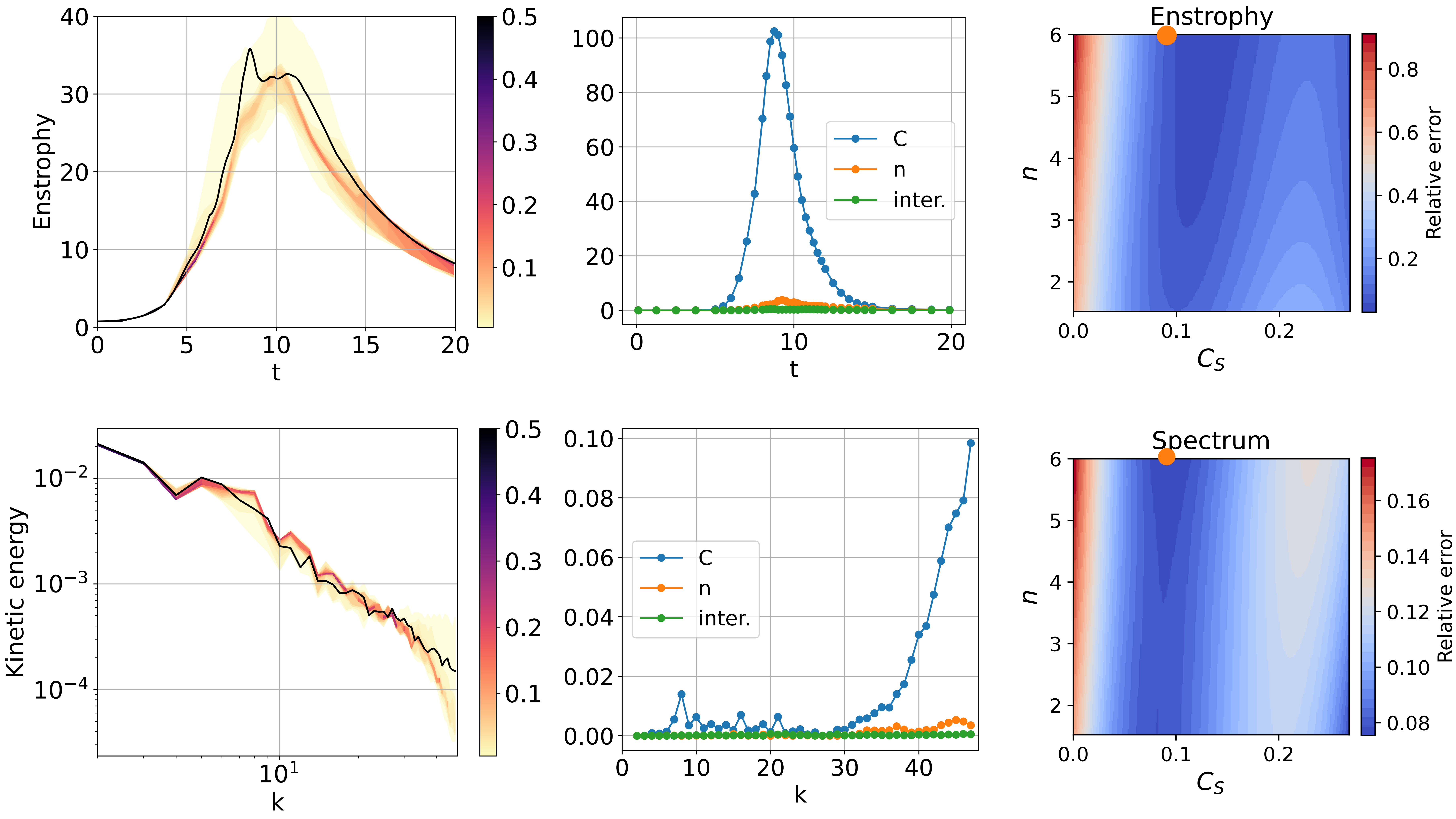
- UQ for **SVV** and **VMS-Smag** on Taylor-Green Vortex at $Re = 5000$ with a resolution of $N_{LES}^3 = 96^3$
- Influence of filter order and model coefficient on enstrophy and kinetic energy spectrum at t=8.5, comparison with DNS (solid lines)



Results for SVV

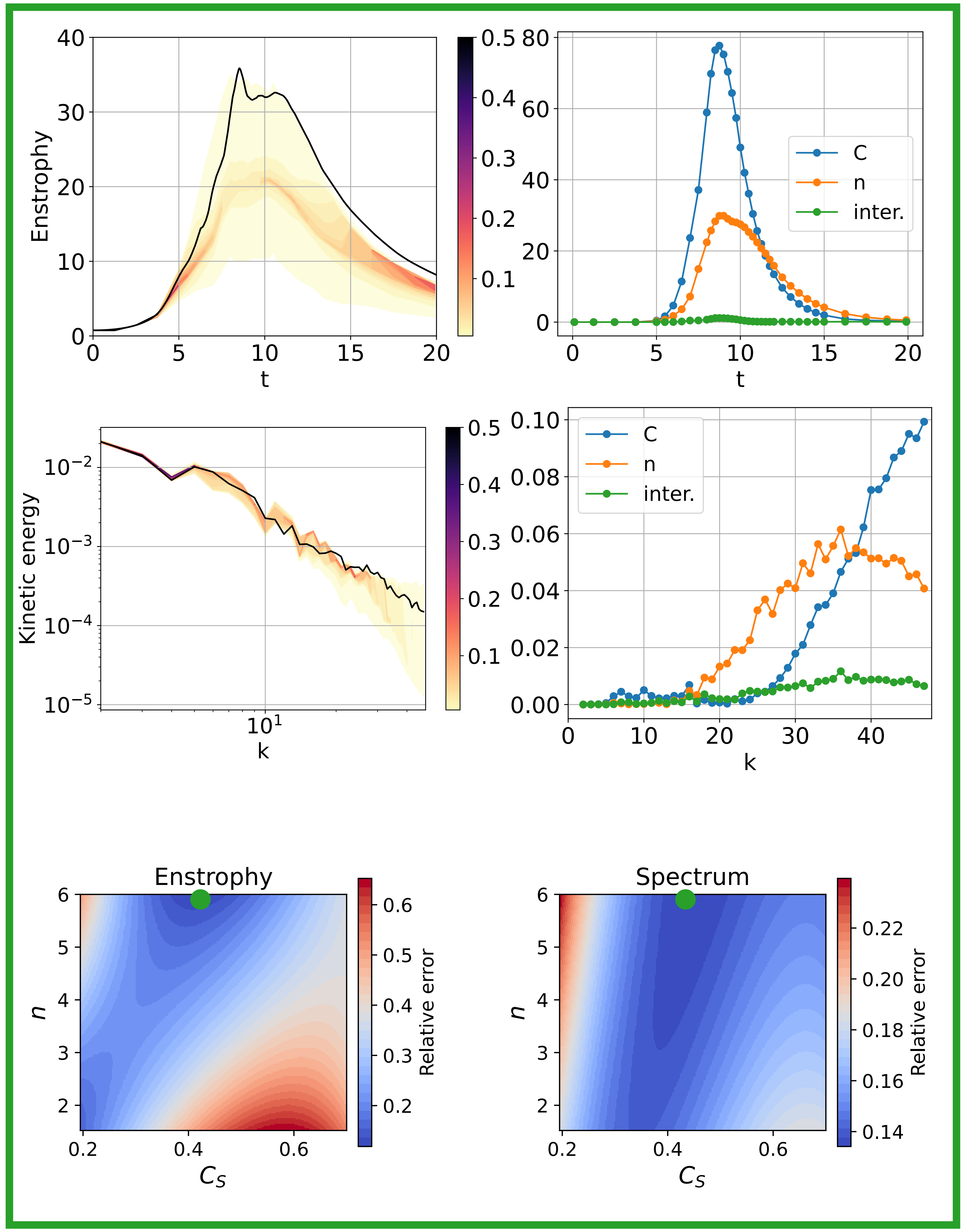
- UQ for **SVV** and **VMS-Smag** on Taylor-Green Vortex at $Re = 5000$ with a resolution of $N_{LES}^3 = 96^3$

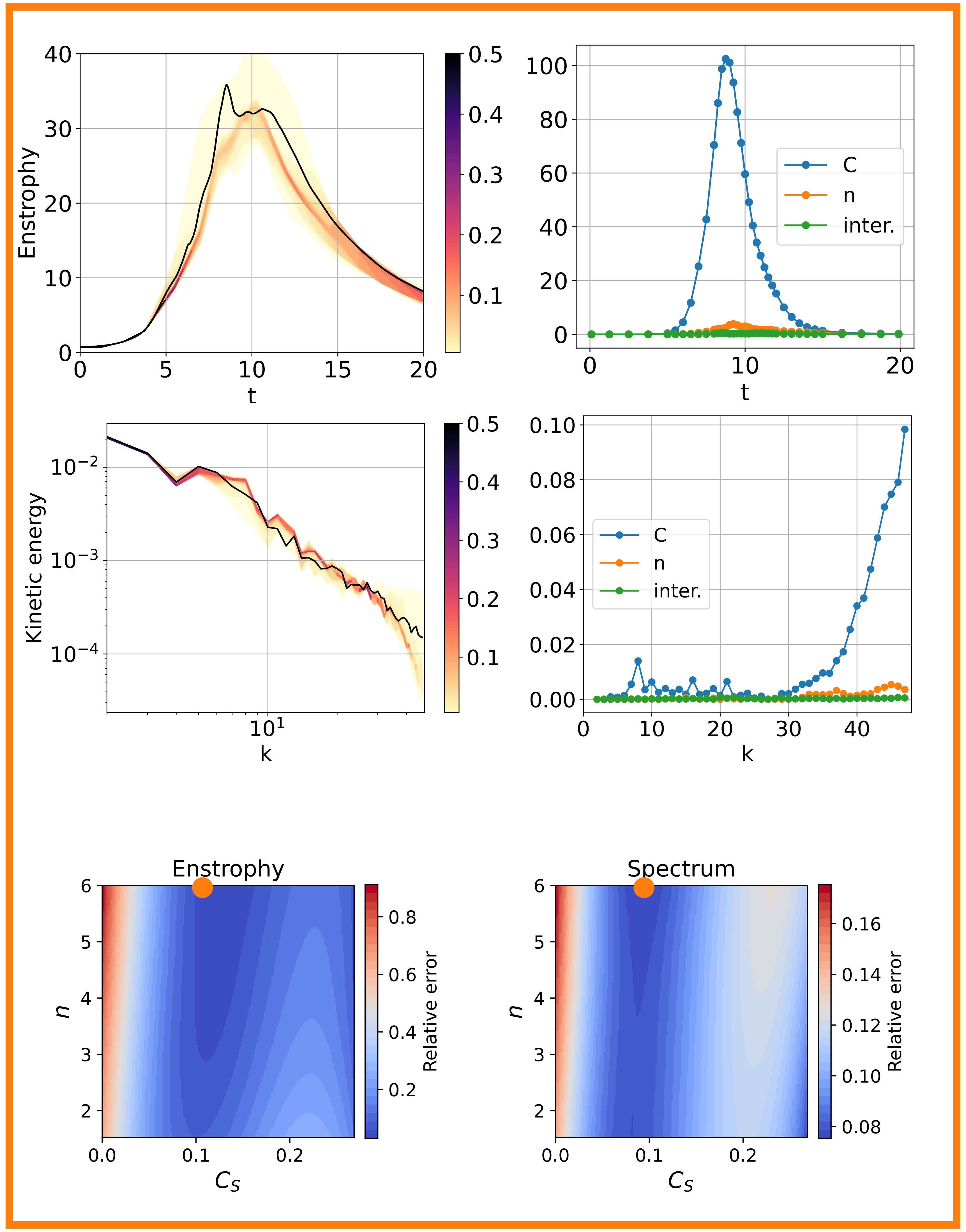
- Influence of filter order and model coefficient on enstrophy and kinetic energy spectrum at t=8.5, comparison with DNS (solid lines)



Results

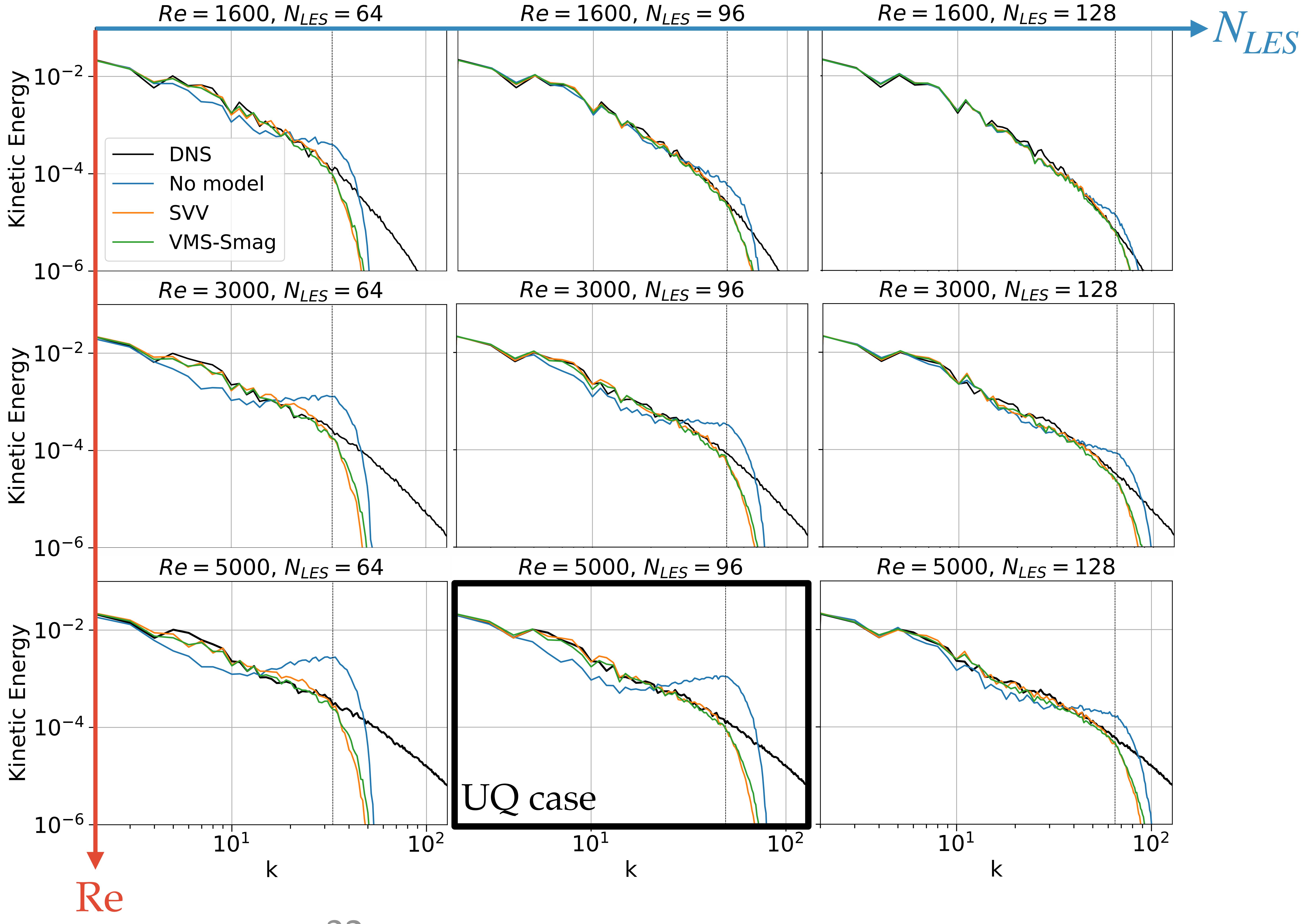
- UQ for **SVV** and **VMS-Smag** on Taylor-Green Vortex at $Re = 5000$ with a resolution of $N_{LES}^3 = 96^3$
- Influence of filter order and model coefficient on enstrophy and kinetic energy spectrum at t=8.5, comparison with DNS (solid lines)
- VMS-Smag results more dispersed than SVV
- SVV not very dependent on the order of the filter used





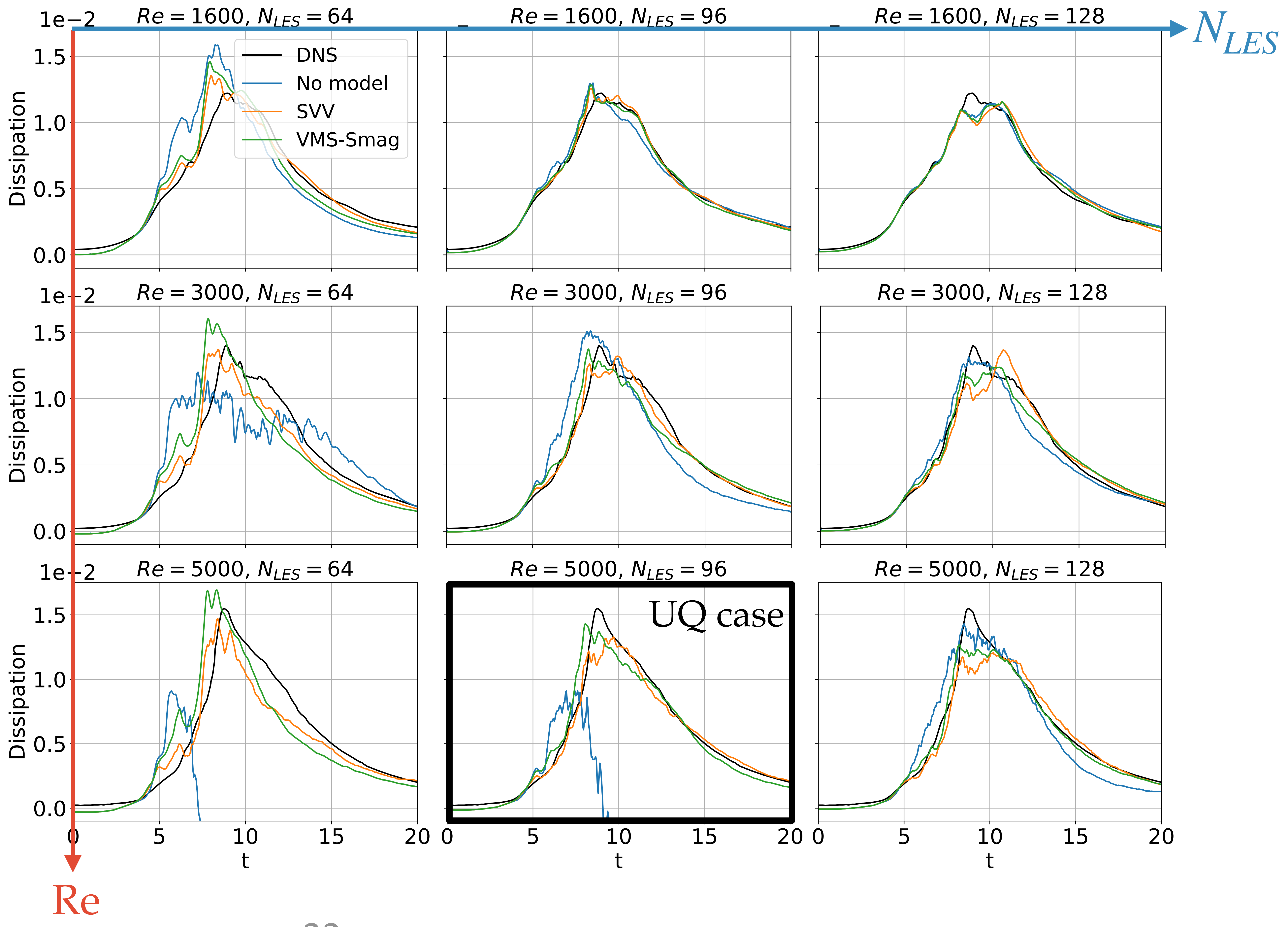
Results

- Coefficients for VMS-Smag and SVV are optimized for $Re = 5000$ and $N_{LES} = 96$
- Optimized coefficients are robust to Reynolds number and resolution variation



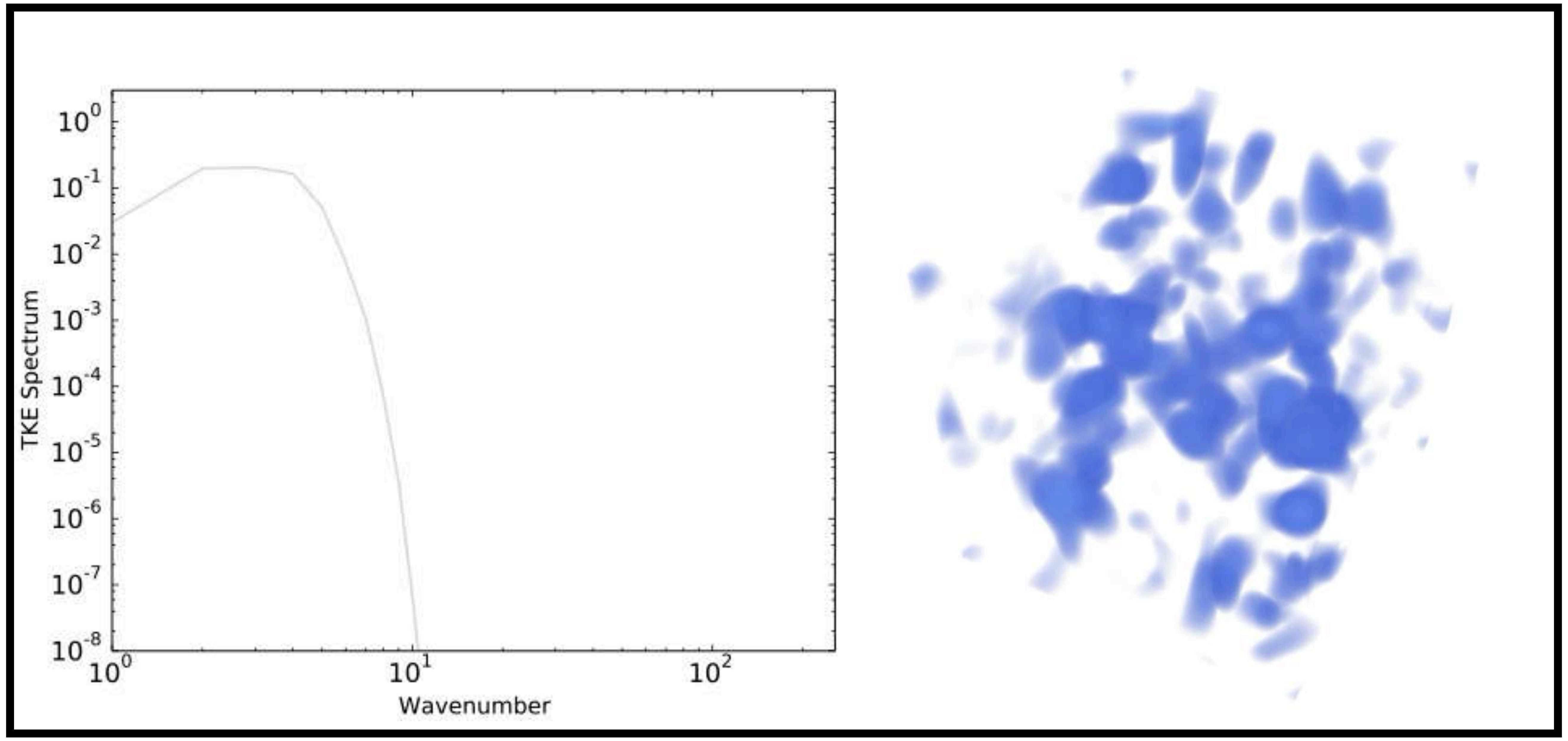
Results

- Coefficients for VMS-Smag and SVV are optimized for $Re = 5000$ and $N_{LES} = 96$
- Optimized coefficients are robust to Reynolds number and resolution variation



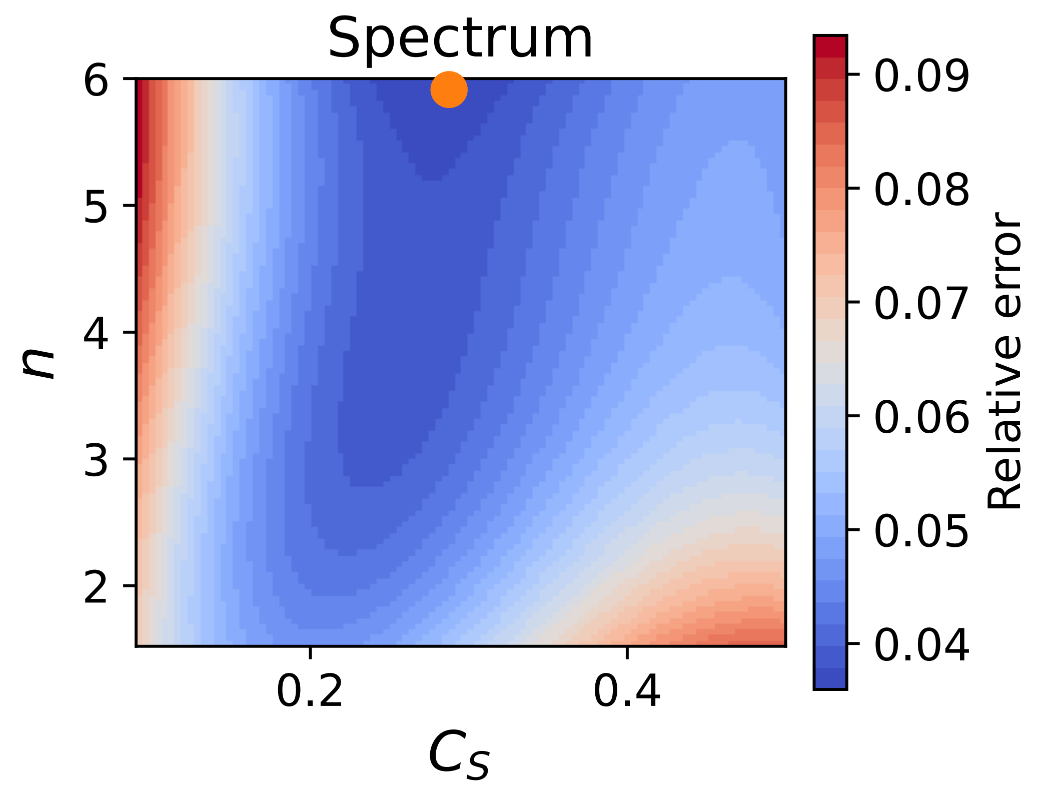
Decay of HIT

- Periodic cubic domain
- Initialization on a high-resolution: turbulence grows through a forcing term
- Converged solution filtered to a coarse resolution to start LES
- Comparison with present DNS

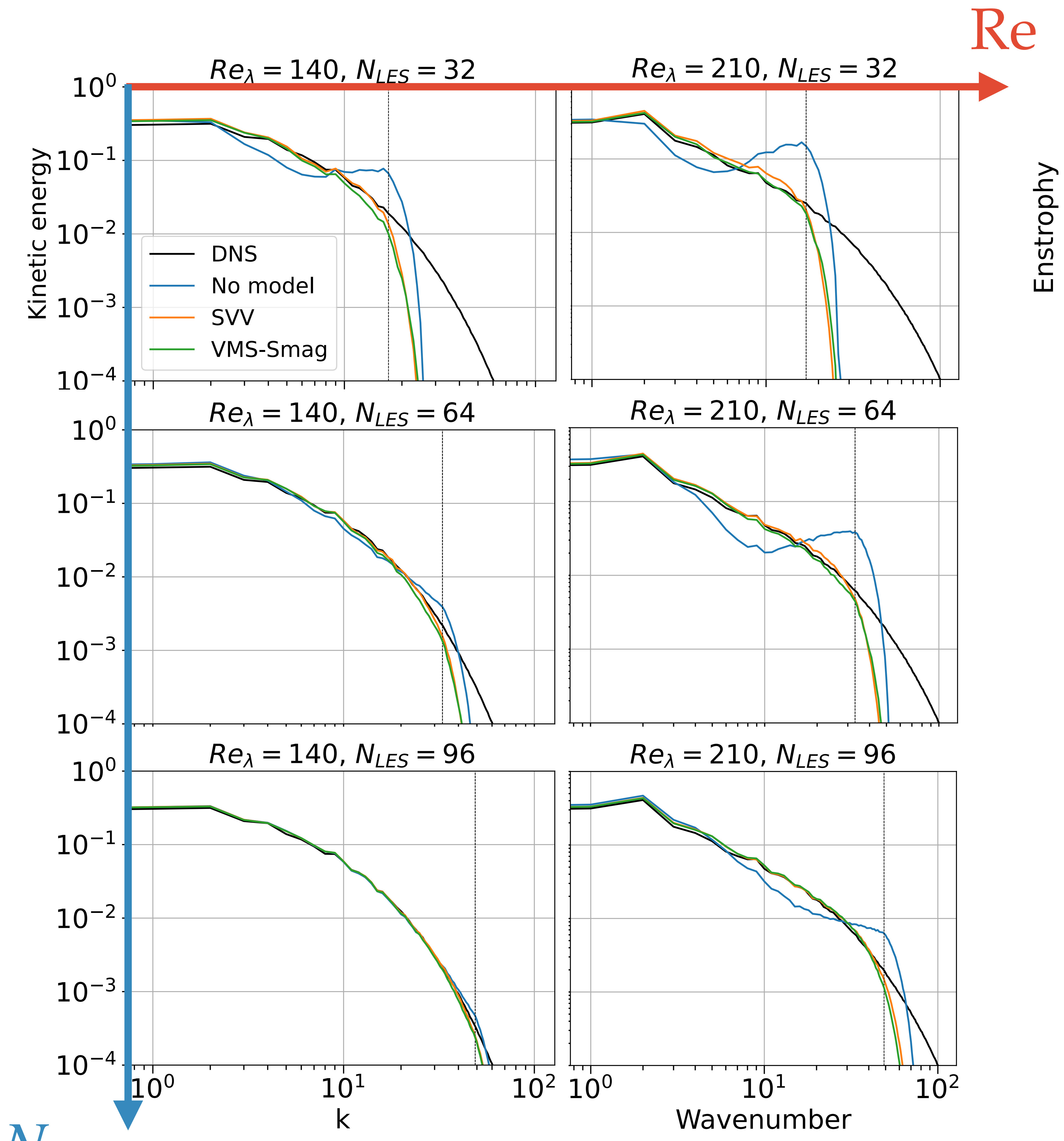


Homogeneous Isotropic Turbulence Model calibration

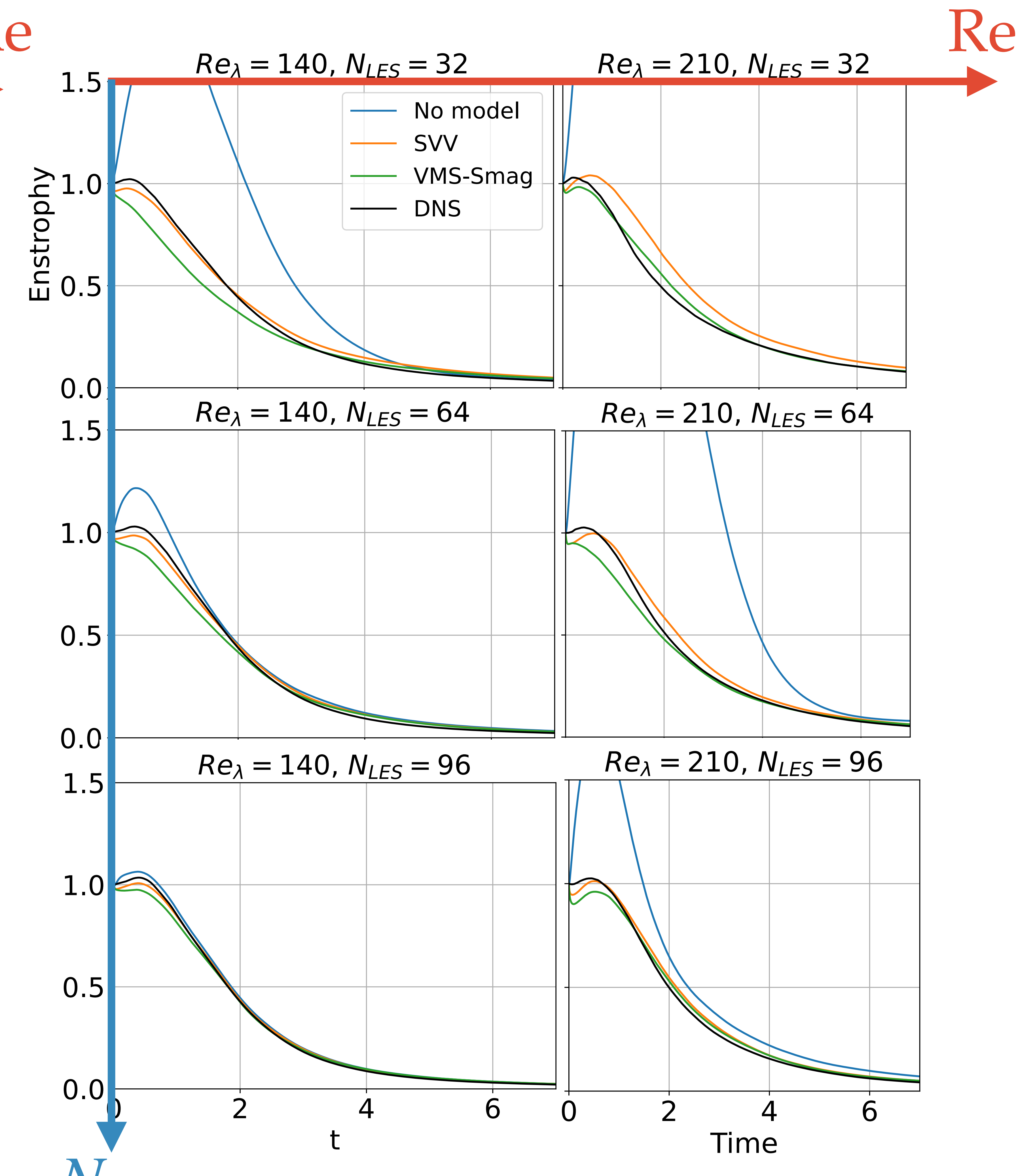
- Coefficients for VMS-Smag are the same as those optimized on TGV
- SVV coefficients required a new calibration



- Optimized coefficients are robust to Reynolds number and resolution variation



Kinetic energy spectrum



Enstrophy

Takeaways

Evaluation of various subgrid scale models for the **TGV benchmark at Re=5000**

- The **VMS-Smag** and **SVV** models were found to be well suited to the present RVM
  - Based on small-scale filtering
  - **SVV** (in Fourier space) especially cost efficient in our algorithm

=> Selected for calibration

- Study of sensitivity of models to parameters using **Polynomial Chaos Expansion**
  - Coefficient had a greater impact than test filter order
  - Calibrated coefficients robust to Re and resolution
  - **VMS** robust to a change in configuration

Context

Methods

Remeshed Vortex Methods

Subgrid-Scale Modeling

Homogeneous Isotropic Turbulence

Model benchmark

Model calibration

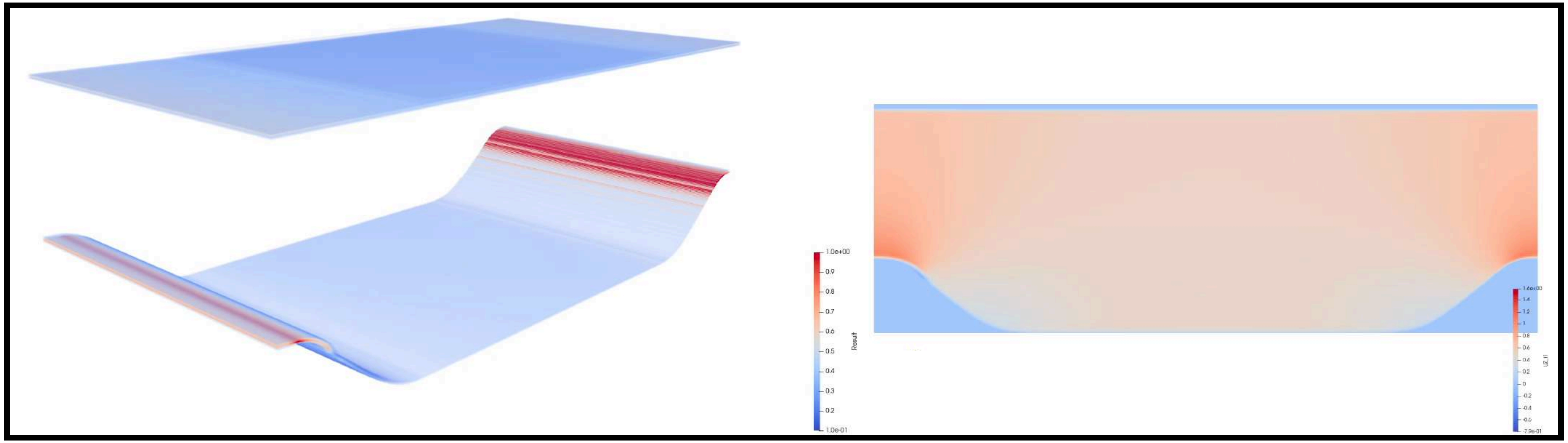
Wall-bounded flows

Flow over periodic hills

Flow past a sphere

Wall-bounded flows Flow over periodic hills

- Classical turbulent flow configuration well studied in literature*
- Flow separated from curved surface and reattached
- Periodic boundary conditions in streamwise and spanwise directions, no slip condition at the upper and lower walls
- Reynolds number $Re_b = U_b h/\nu$ computed from bulk velocity and hill height, here Re=2800 (DNS) or Re=10 595 (LES)



*Mellen et al. (2000), Temmerman and Leschziner (2001), Fröhlich et al. (2005), Breuer et al. (2009), Rapp and Manhart (2011), Krank et al. (2018), Gloerfelt and Cinnella (2019), Wang et al. (2021)…

Numerical Setup

- **Solid boundaries** treated with a **penalization method*** (**Brinkman penalization**)

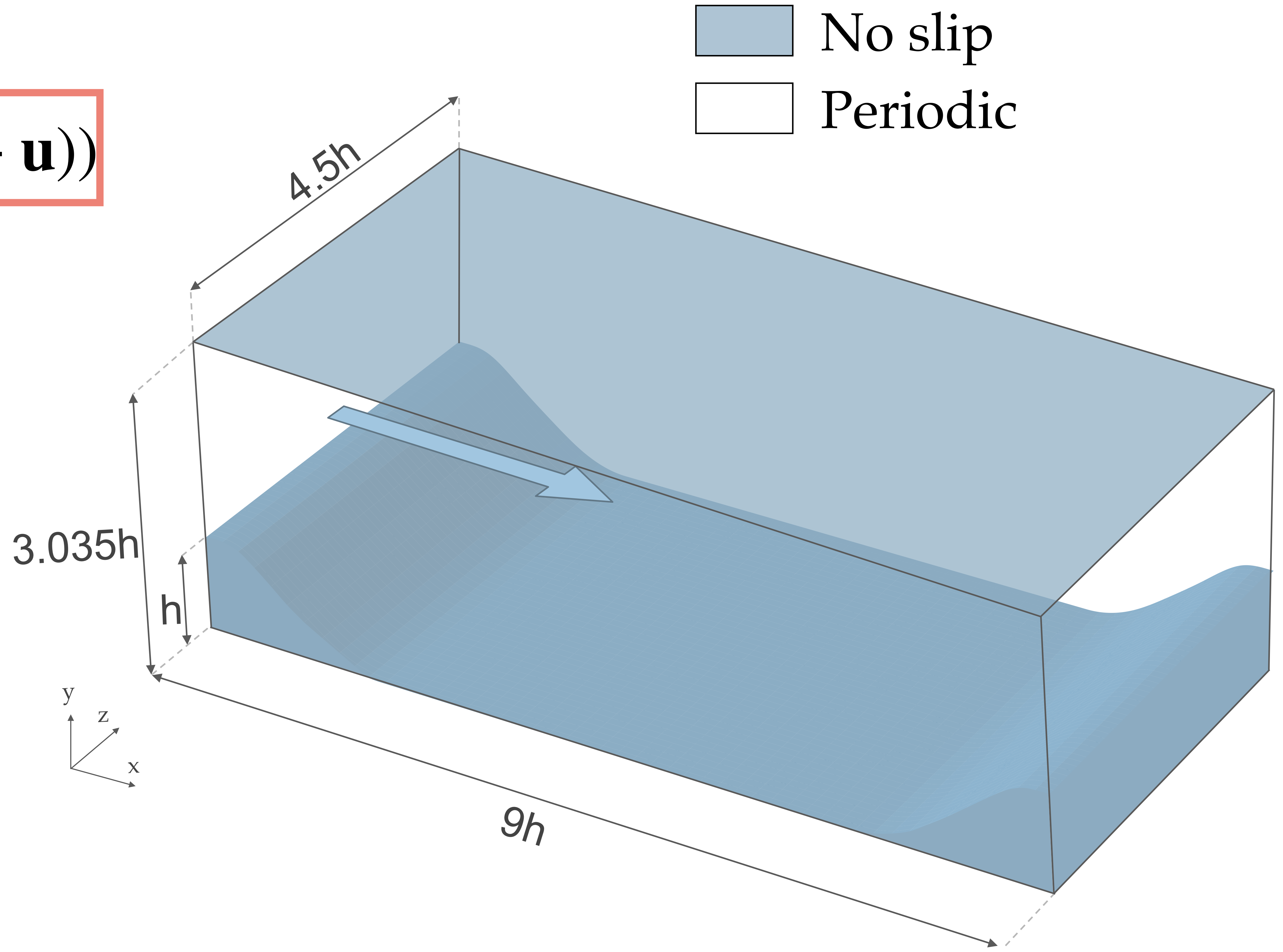
$$\frac{\partial \omega}{\partial t} + \mathbf{u} \cdot \nabla \omega - \omega \cdot \nabla \mathbf{u} - \frac{1}{Re} \Delta \omega = \boxed{\nabla \times (\lambda_\chi(\mathbf{u}_S - \mathbf{u}))}$$

  - Forcing term where permeability λ_χ depends on Darcy's law

- **Time- and spanwise-averaged statistics**

  - Averaging after 20 flow-throughs

  - Averaging time $\Delta T_{av} = 60$ flow-throughts

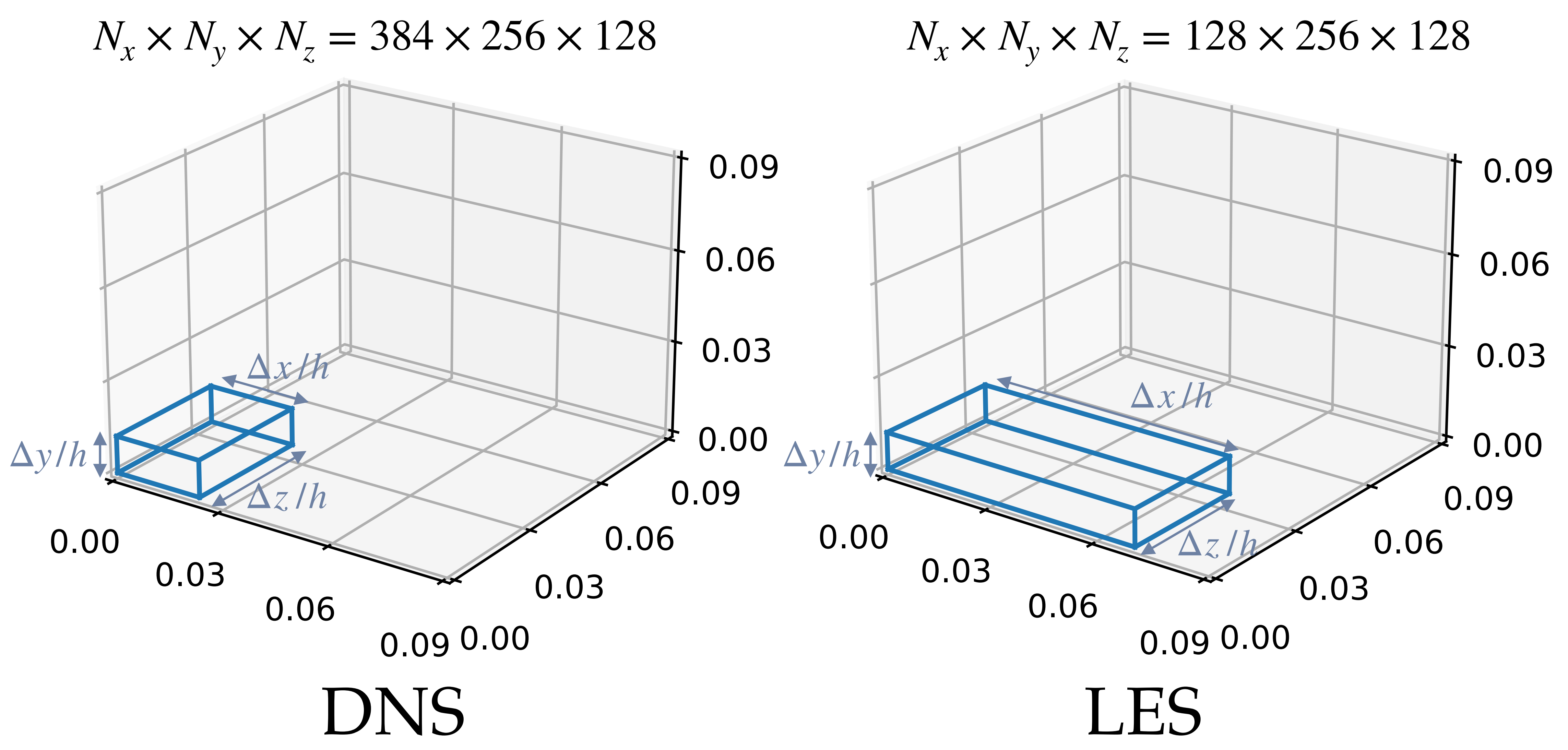


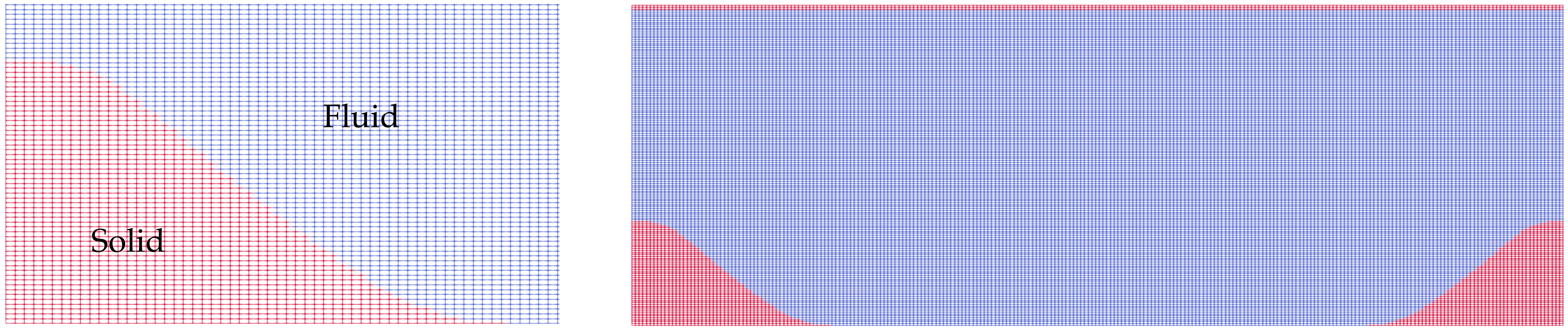
*Caltagirone (1994), Angot et al. (1999), Kevlahan and Ghidaglia (2001), Coquerelle and Cottet (2008), Mimeau et al. (2017)

Numerical Setup

Cartesian grids uniform by direction

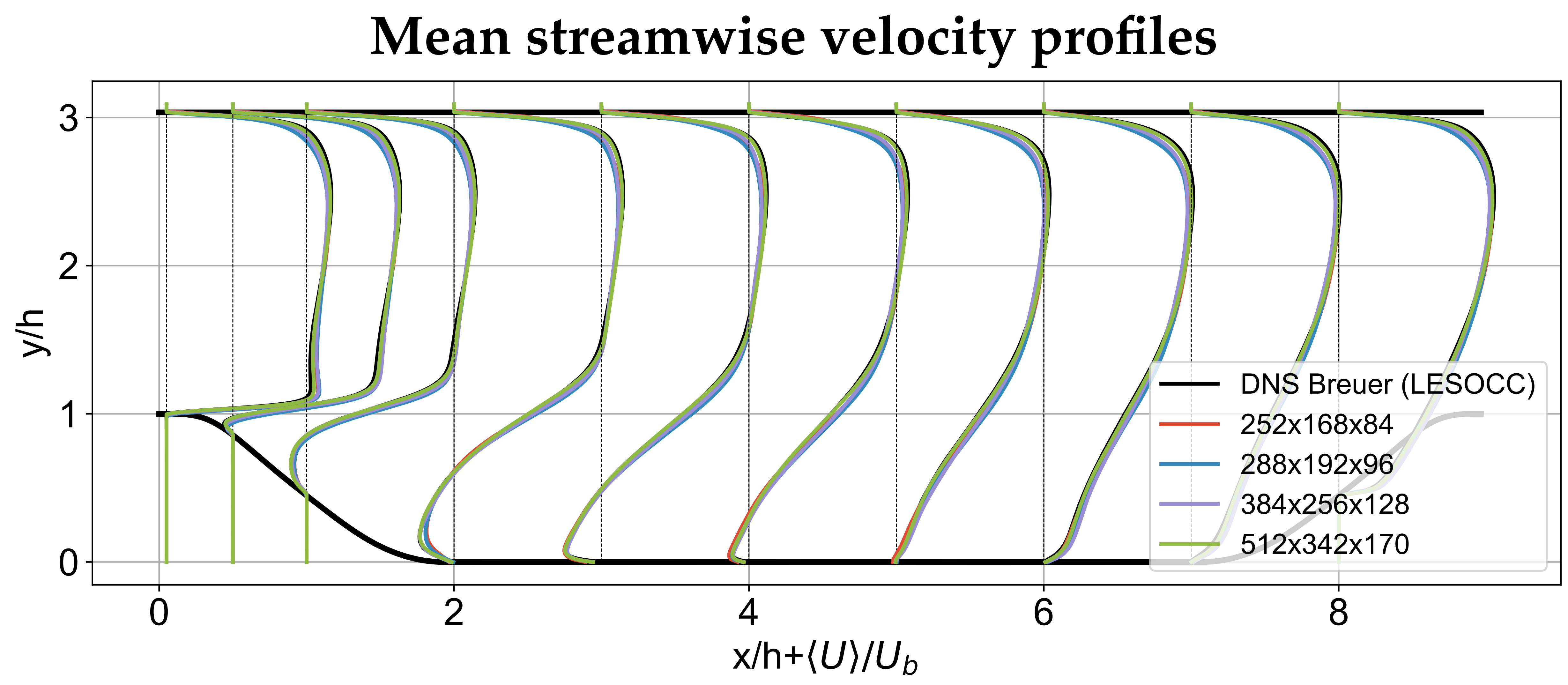
- Rectangular cells with AR vertical / spanwise = 3
  - For DNS (Re=2800) : AR vertical / streamwise = 2
  - For LES (Re=10 595) : AR vertical / streamwise = 6





DNS results for Re=2800 (mean flow)

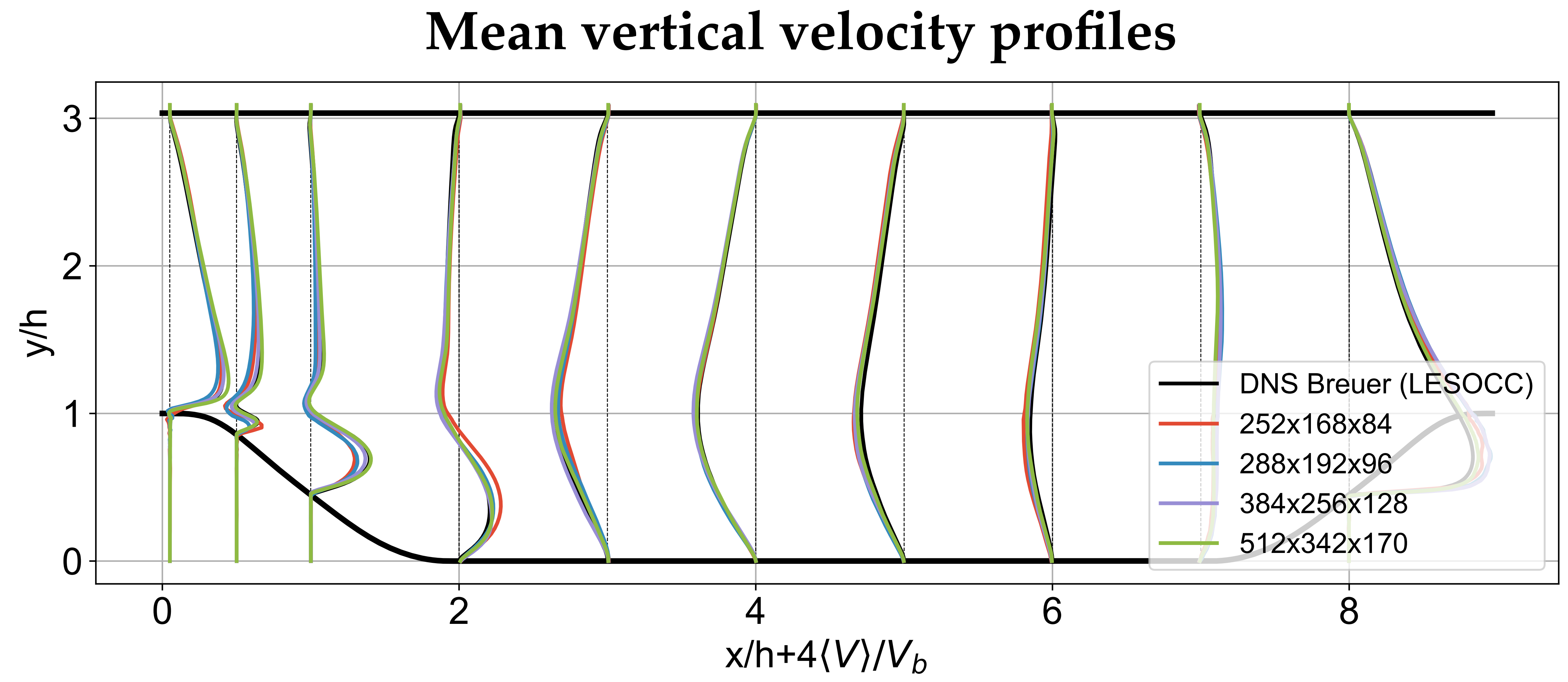
- Grid convergence study :
  - Grid 1 : 252x168x84
  - Grid 2 : 288x192x96
  - Grid 3 : 384x256x122
  - Grid 4 : 512x342x170



- Comparison with DNS from Breuer et al. (2009)

- Between 1st and 2nd order convergence

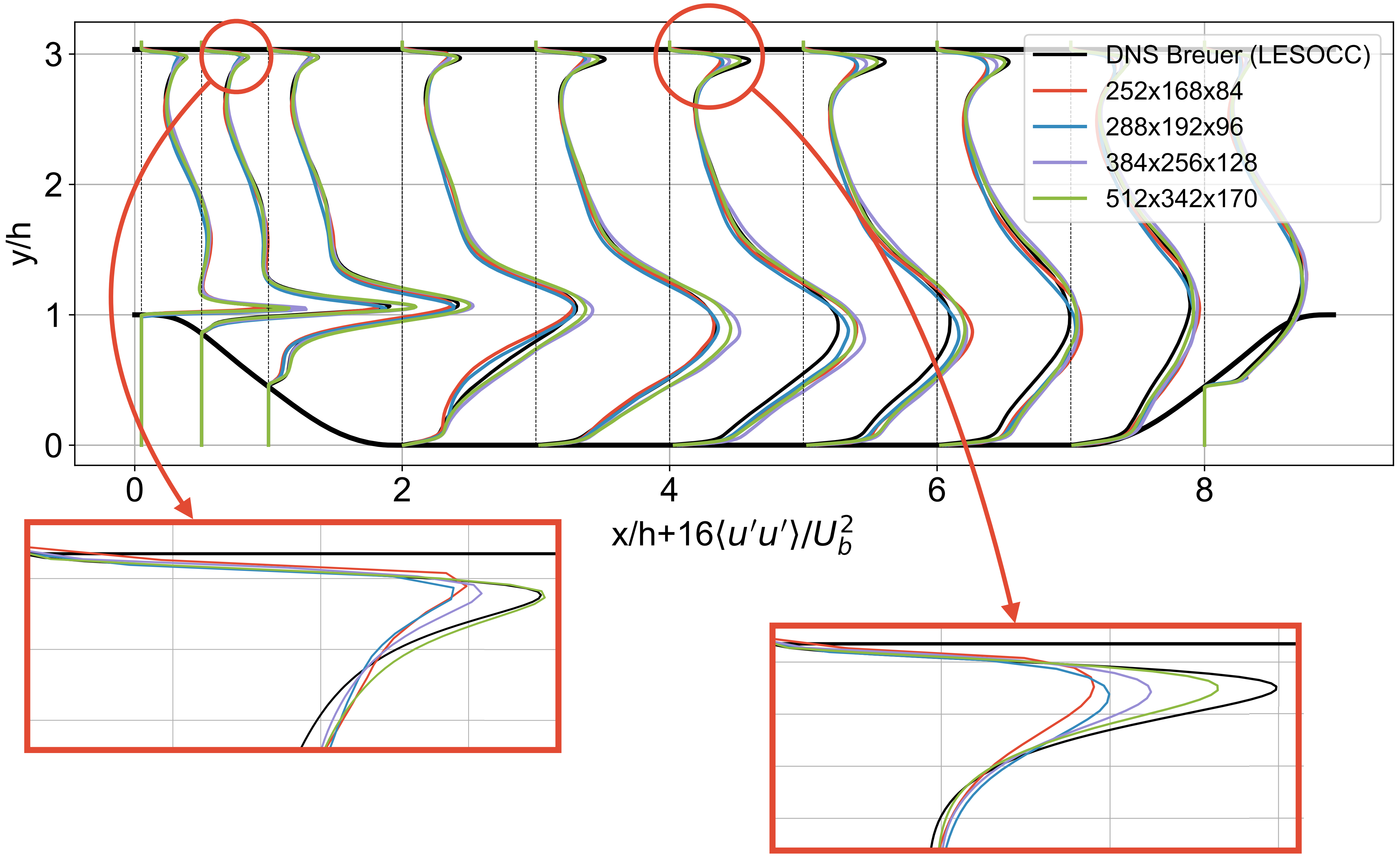
- Better convergence and precision for streamwise velocity



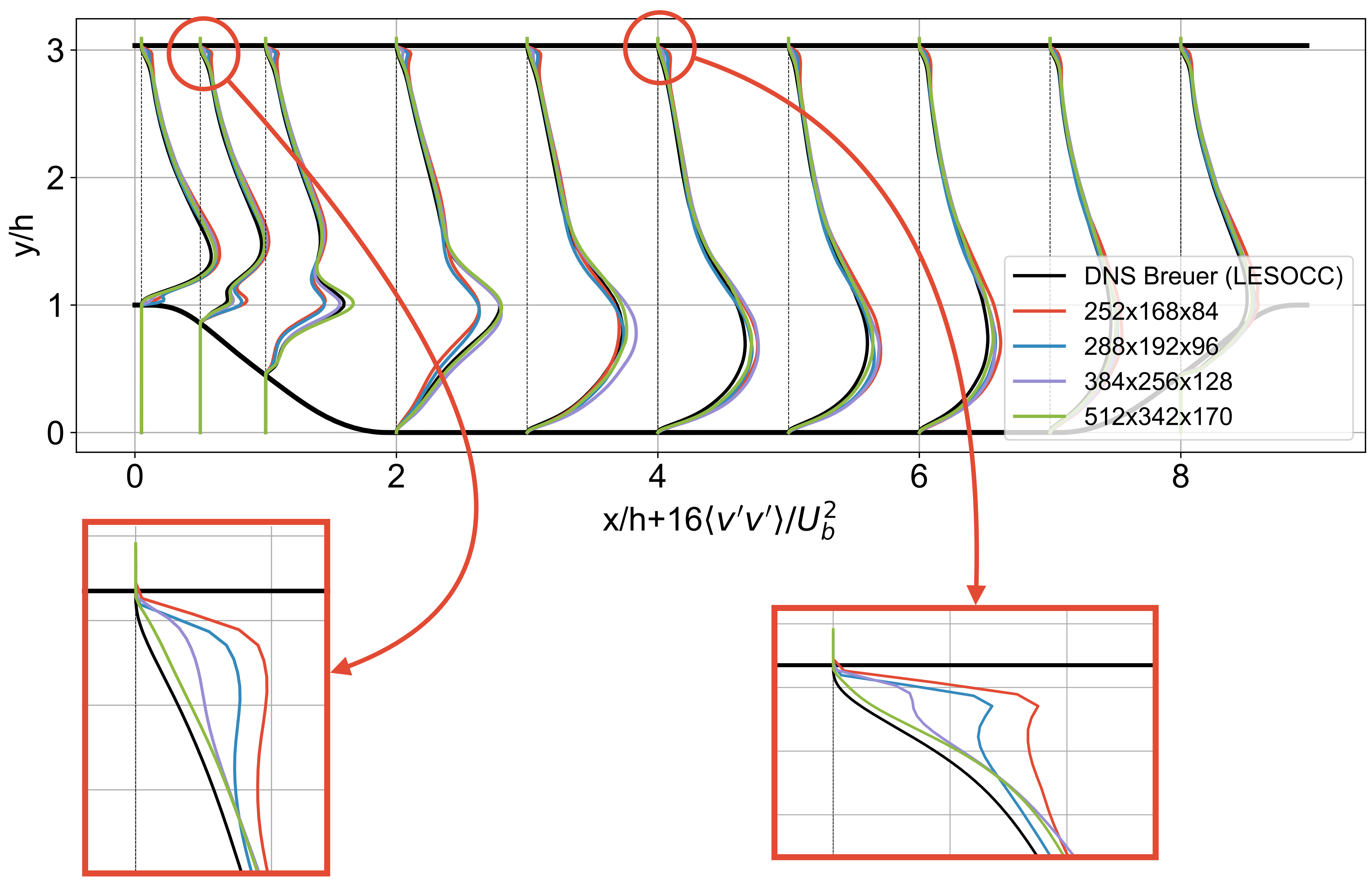
DNS results for Re=2800 (Reynolds stresses)

Good performance for Reynolds stresses near walls despite Cartesian grid

Mean streamwise Reynolds normal stress profiles



Mean vertical Reynolds normal stress profiles



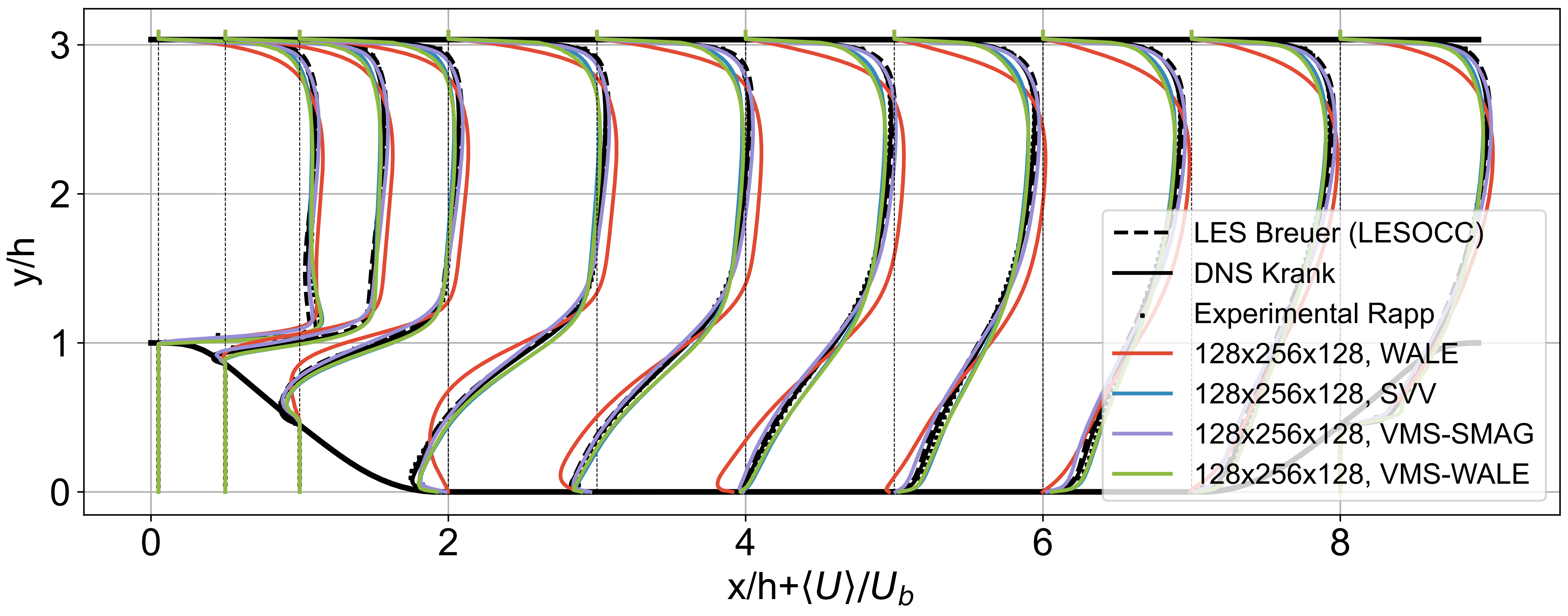
Comparison with Breuer et al. (2009)

LES results for Re=10 595 (mean flow)

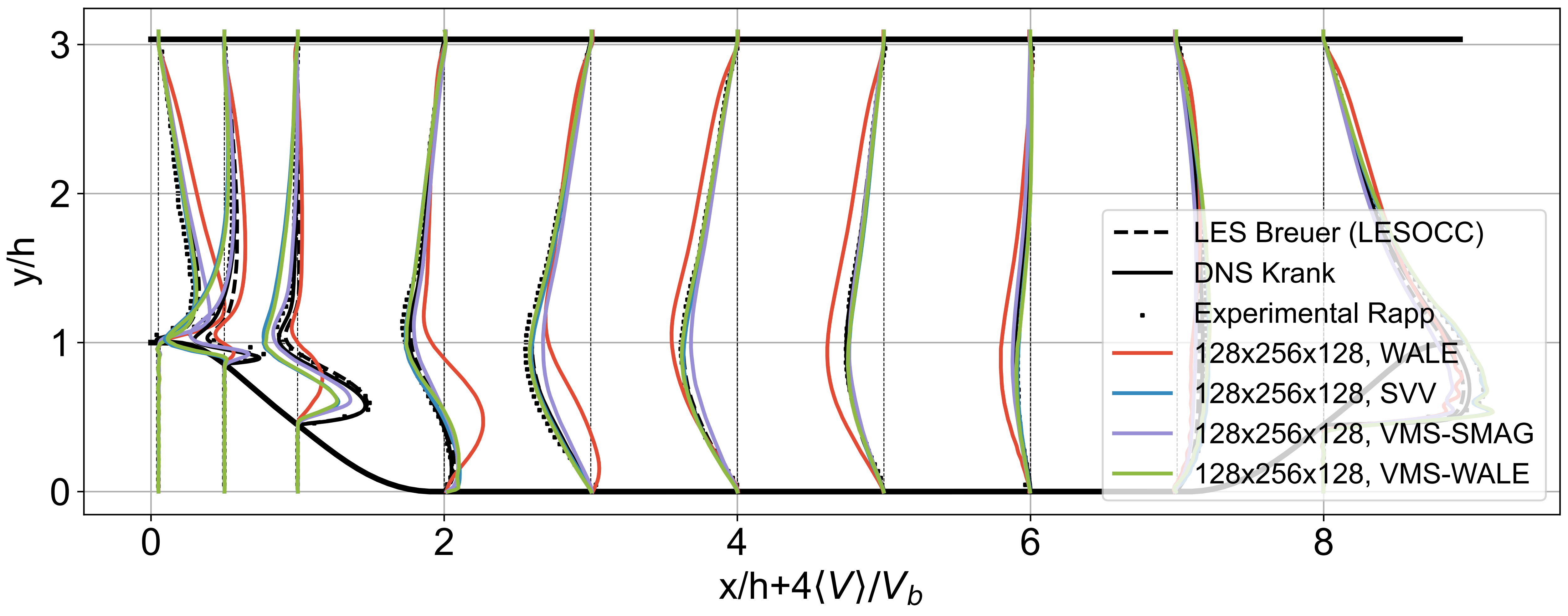
SGS models comparison on 128 x 256 x 128 grid

- Previous two models (VMS-Smag and SVV) with parameters optimized on TGV test case
- WALE and VMS-WALE (using same explicit filter as VMS-Smag)
- Reference LES (Breuer et al. (2009)), DNS (Krank et al. (2018)) and experimental data (Rapp and Manhart (2011))

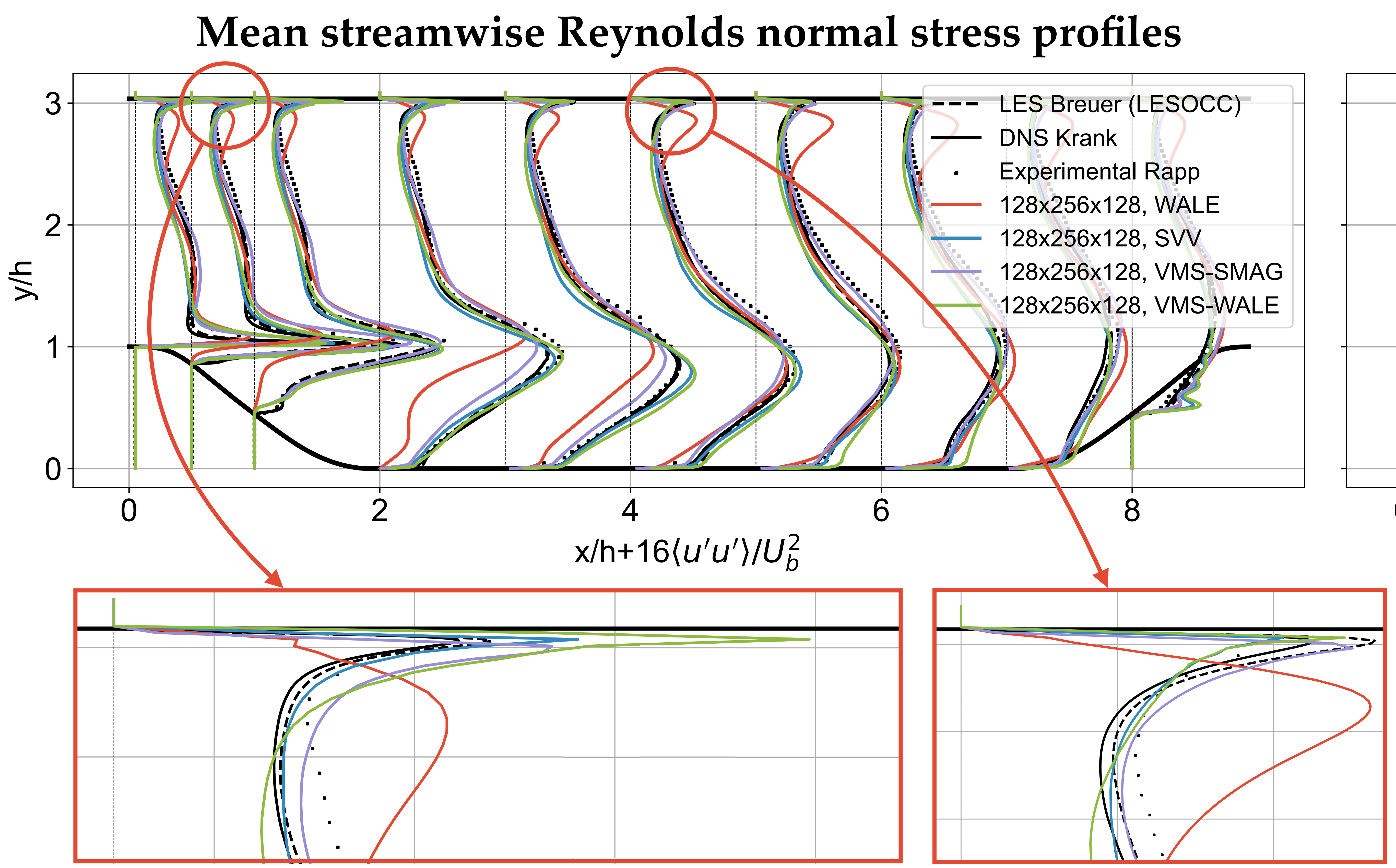
Mean streamwise velocity profiles

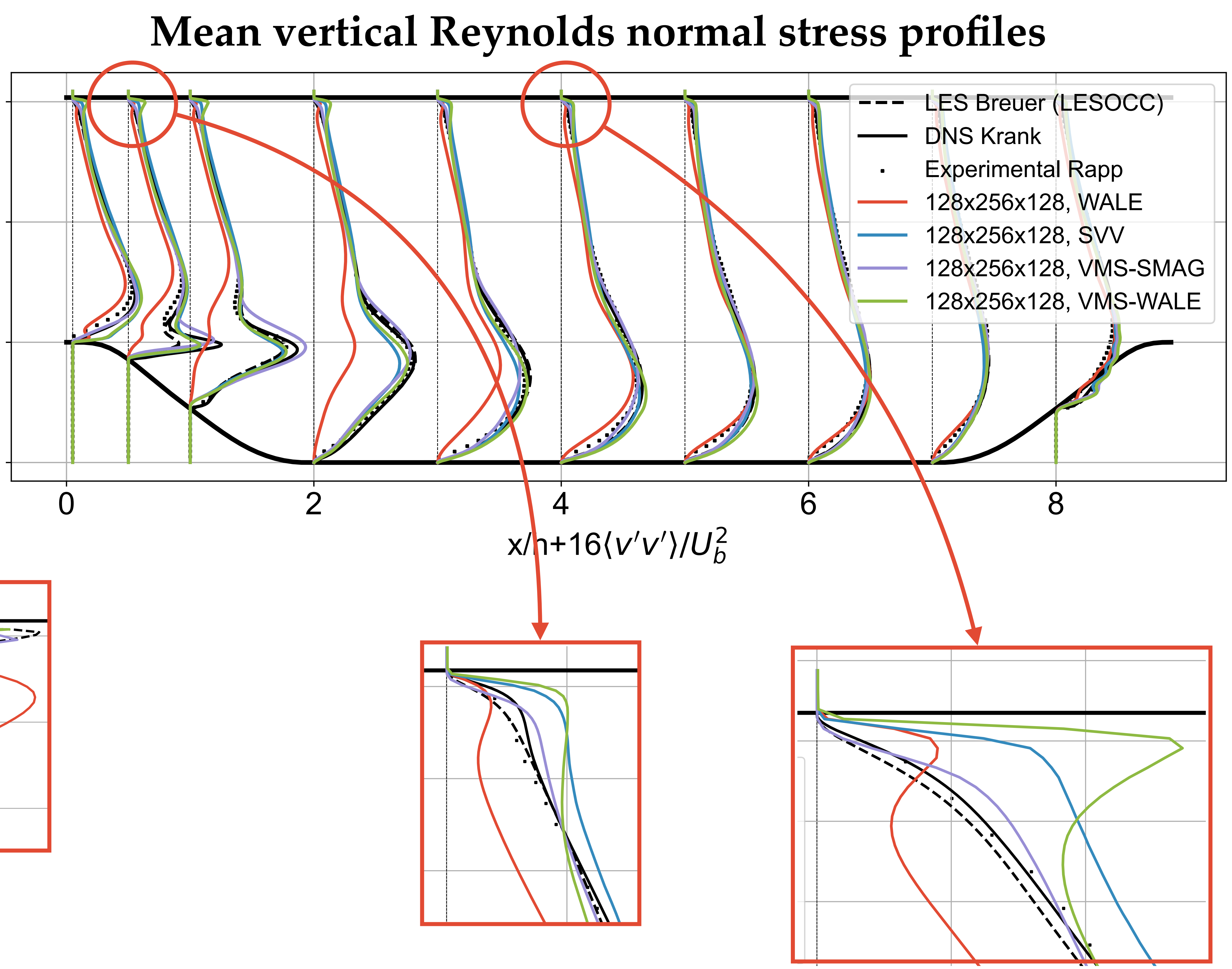


Mean vertical velocity profiles



LES results for Re=10 595 (Reynolds stresses)



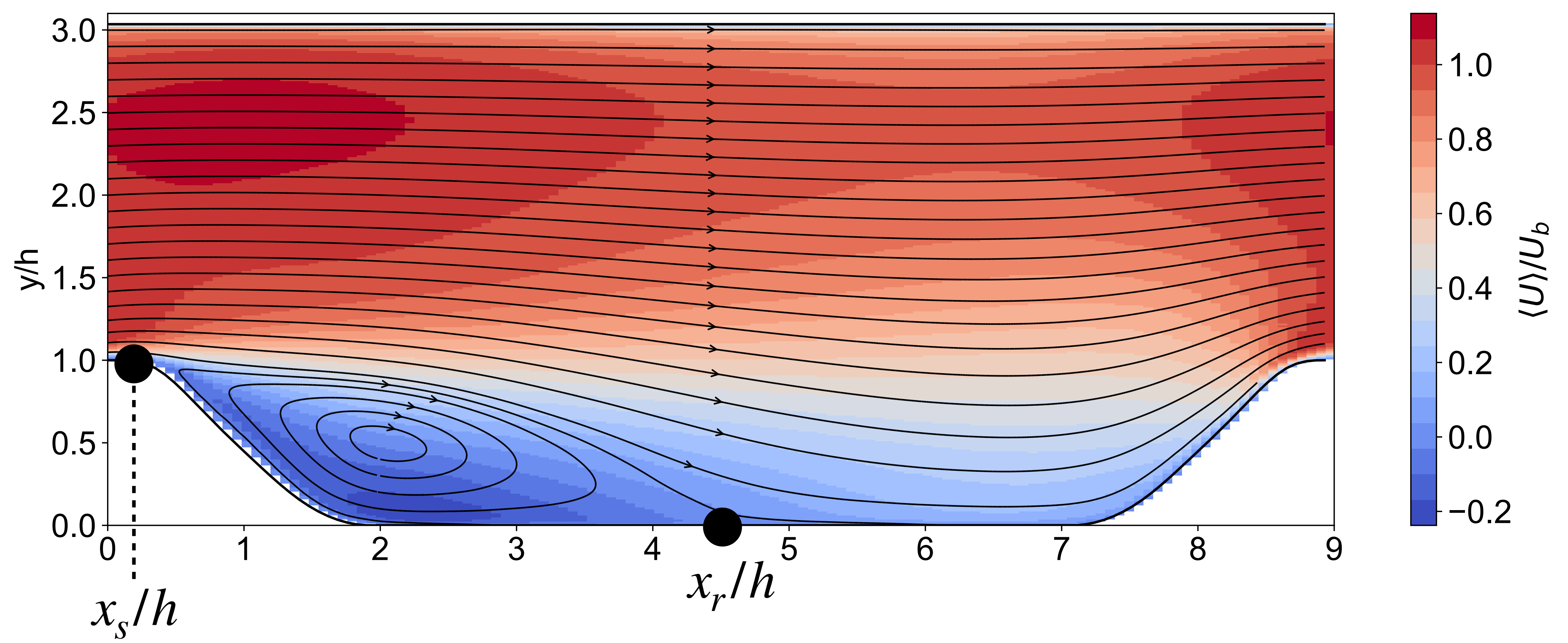


Comparison with Breuer et al. (2009), Krank et al. (2018), Rapp and Manhart (2011).

LES results for Re=10 595

	Separation point x_s/h	Reattachment point x_r/h
Breuer et al. (LES)	0.19	4.69
Krank et al. (DNS)	0.2	4.51
Rapp & Manhart	-	4.21
WALE	0.4	5.6
SVV	0.4	3.9
VMS-Smag	**0.2**	**4.5**
VMS-WALE	0.4	3.9

Mean streamwise velocity



- Good results despite coarse Cartesian grid
- WALE simulations perform the worst
- VMS-WALE outperforms WALE, highlighting the benefits of small-scale filtering
- SVV and VMS-Smag both give good results for mean profiles
- Only VMS-Smag was able to recover approximately the separation and reattachment of the recirculation bubble

Context

Methods

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Homogeneous Isotropic Turbulence

Model benchmark

Model calibration

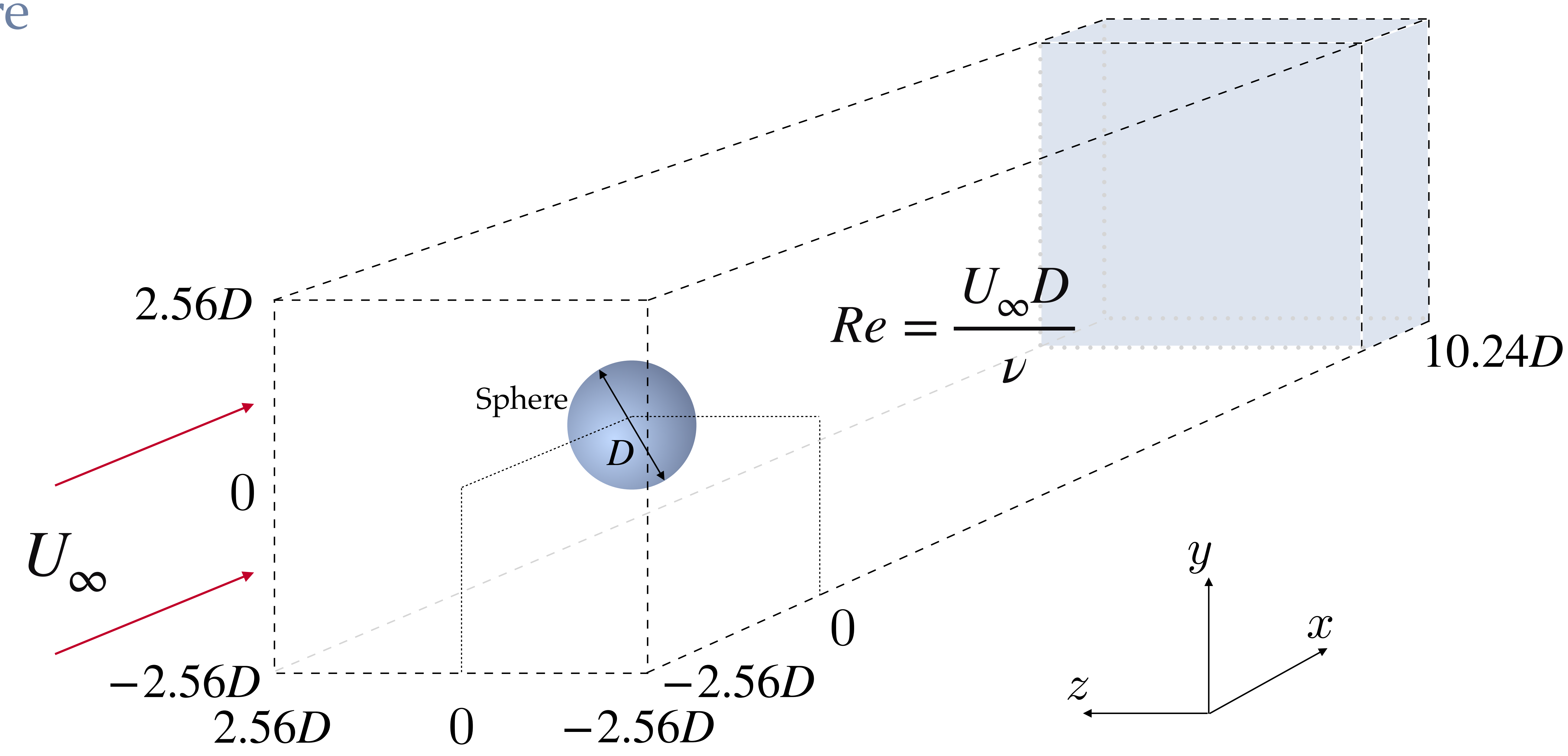
Wall-bounded flows

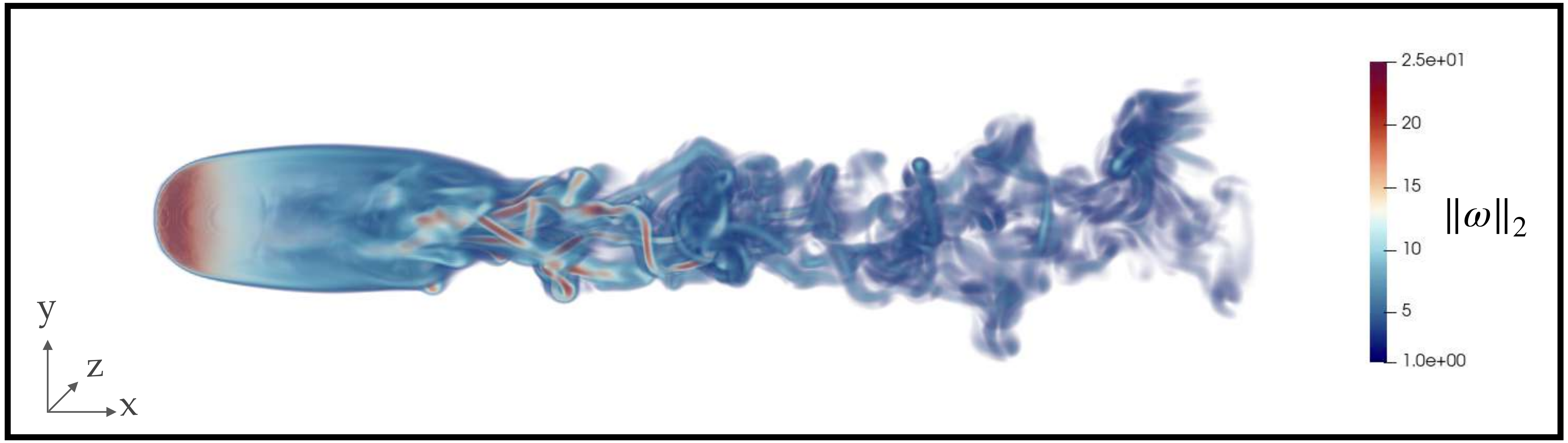
Flow over periodic hills

Flow past a sphere

Setup

- Flow past a solid sphere at Re=3700
- Constant flowrate at entrance
- Vorticity absorption at the end of the domain through a sponge zone
- Comparison with DNS from Rodriguez et al. (2013)





Uncertainty quantification

- New uncertainty quantification for **VMS-Smag**
- Coarse 320x128x128 grid considered

Uncertain parameters

-model coefficient : C_S^2
-filter order : n

Quantity of interest $q(C, n)$ **obtained from LES** (time spectra of fluctuations of radial velocity)

Hysop

Dakota open source software

PCE : given polynomial basis $(\Psi_i)_i$ find coefficients $(\hat{q}_i)_i$ such that

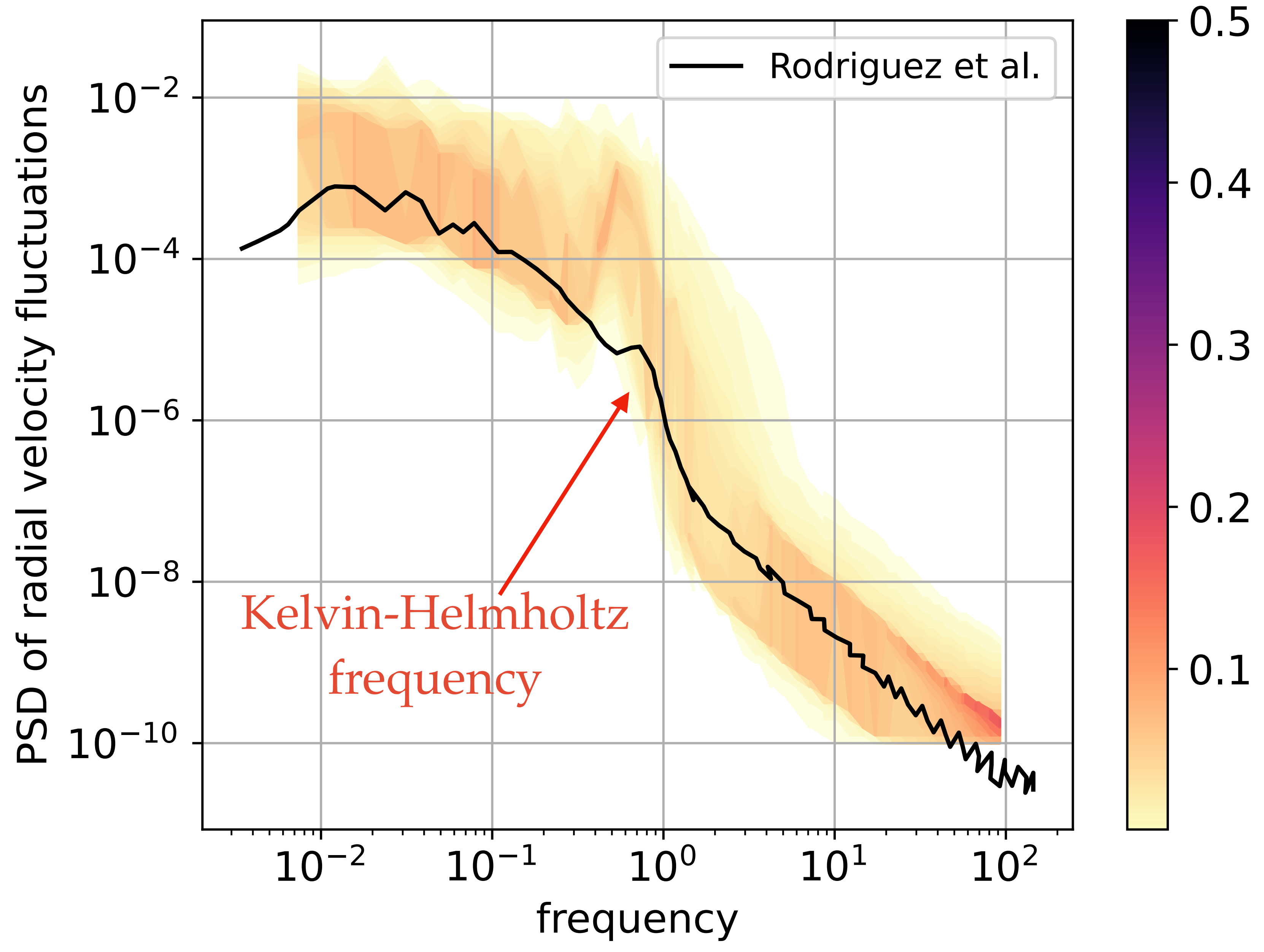
$$q(y_1, \ldots, y_n) = \sum \hat{q}_i \Psi_i(y_1, \ldots, y_n)$$

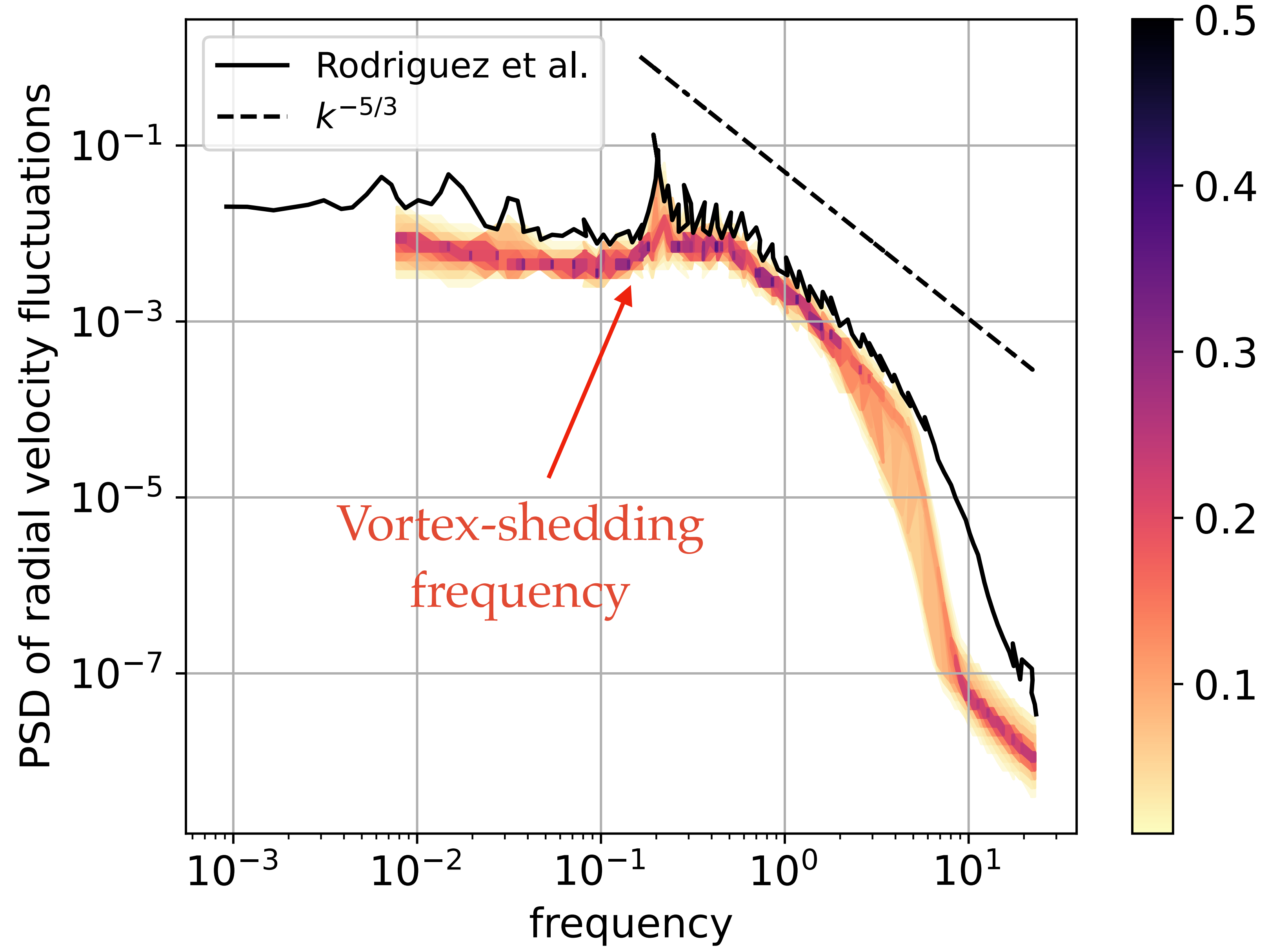
Estimation of mean and variance of $q(y_1, y_2, \ldots, y_n)$

Stochastic calibration : if $q(y_1, y_2, \ldots, y_n)$ is the discrepancy with the reference value we can find coefficients minimizing it

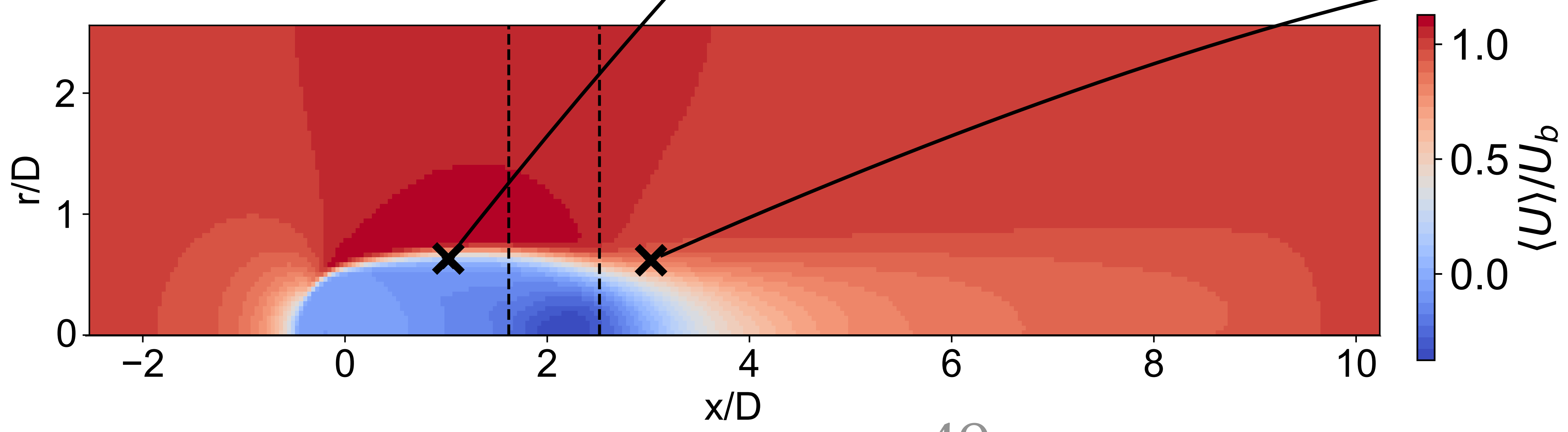
Wall-bounded flows Flow past a sphere

- Model sensitivity dependant on location
- Close to the sphere, the flow is very sensitive to near-wall effects
  - LESs are very sensitive to the model parameters
  - Shift in frequency for Kelvin-Helmholtz frequency compared to reference
- Further downstream, the turbulence is more developed
  - Vortex-shedding frequency recovered
  - The LESs are less sensitive in this area and recover the largest frequencies



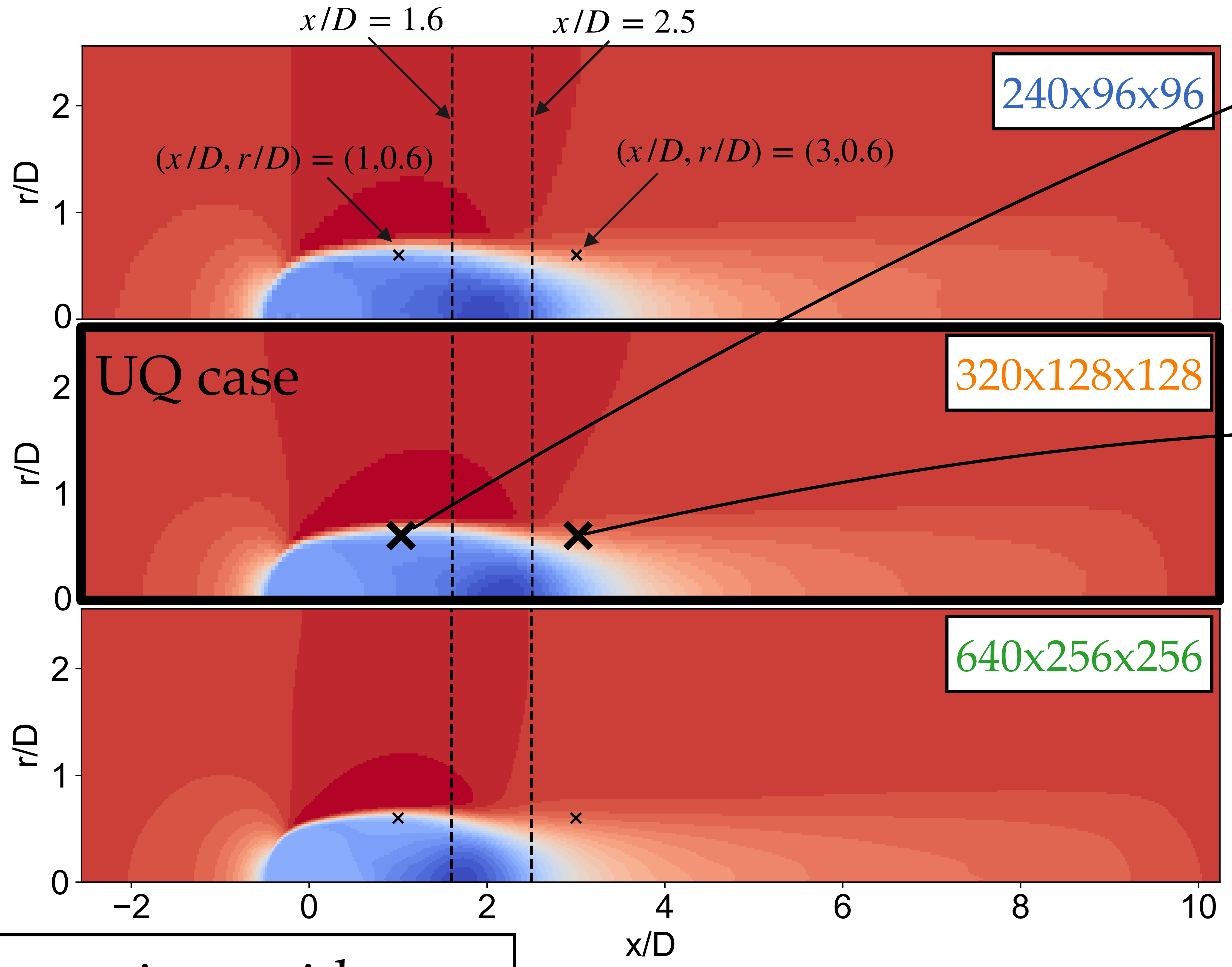


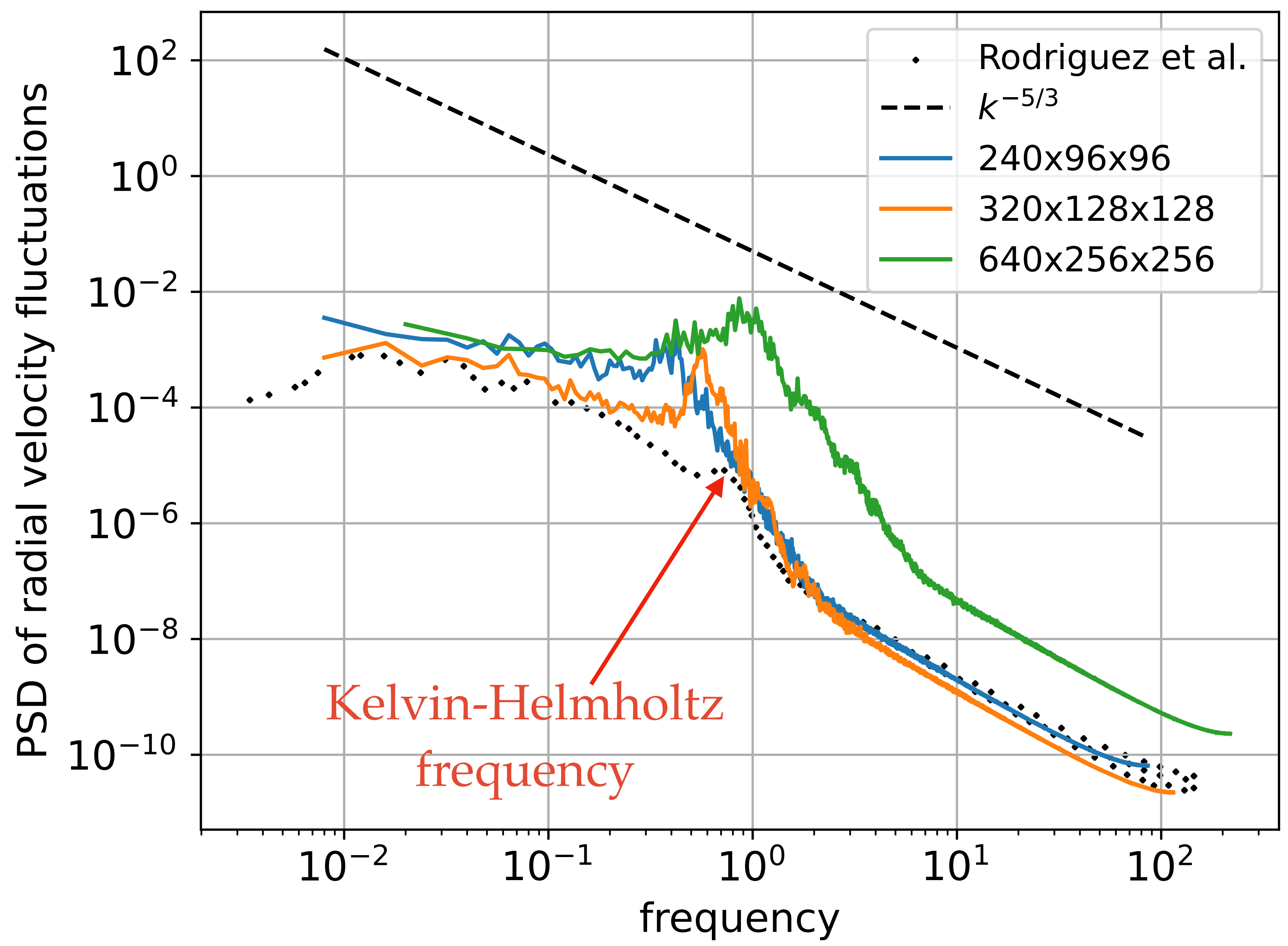
Comparison with Rodriguez et al. (2013)

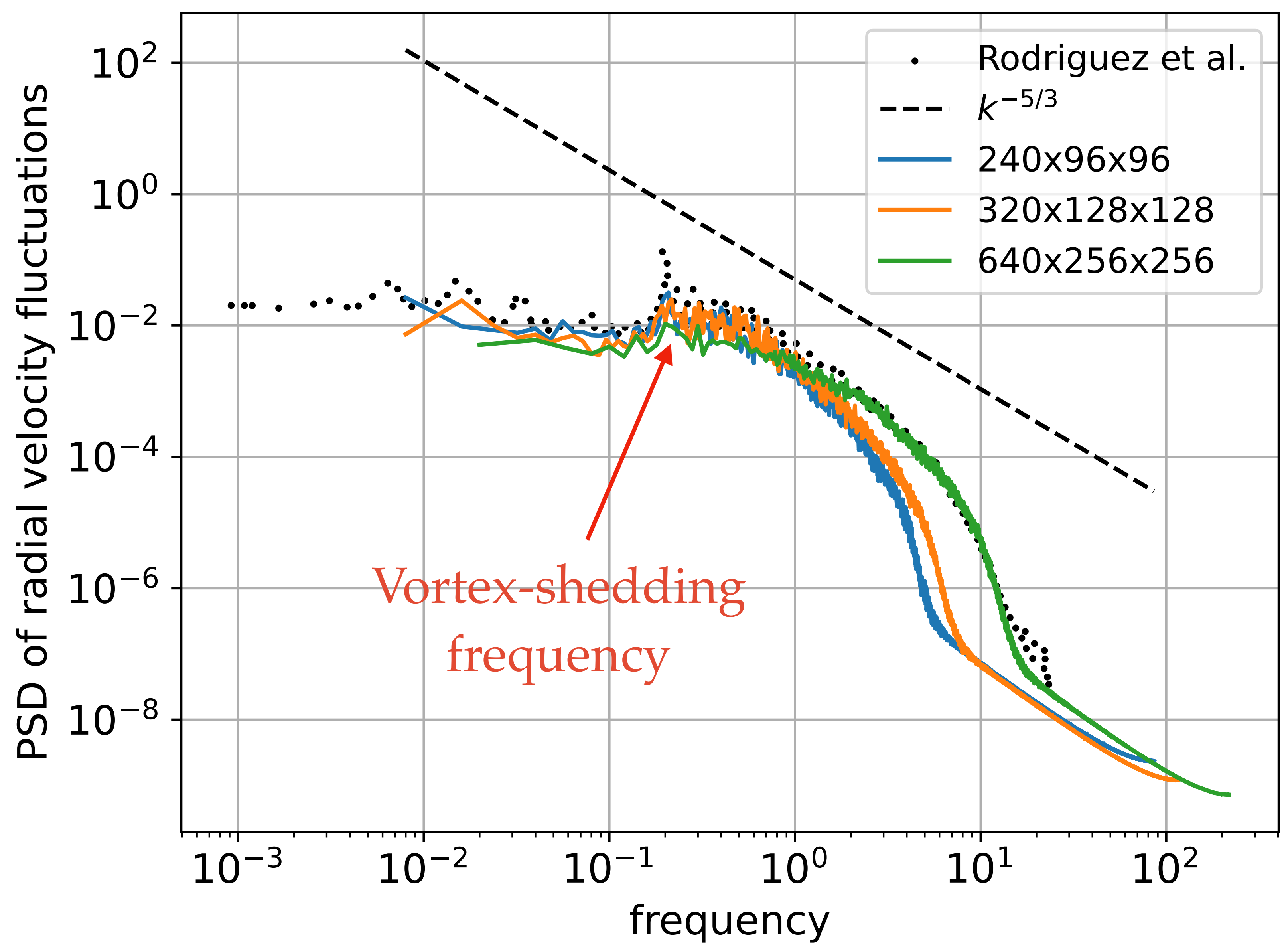


Wall-bounded flows Flow past a sphere

- Model calibration not robust to an increase in **resolution**
- Grid too coarse to properly represent near wall effects







Comparison with Rodriguez et al. (2013)

Takeaways

- Closer downstream the sphere
  - Strong variance of the results with LES coefficients
  - Influence of the grid on Kelvin-Helmholtz frequency and fluctuations amplitude
- Further dowstream the sphere
  - Less influence of LES coefficients and grid
- Difficulty to calibrate models
  - Grid influence : cannot calibrate on coarse resolution
  - More diffusion needed close to the boundary : one set of coefficient for the whole domain might not be enough

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Conclusion

Conclusion

- Best performance found for models targetting small scales : focus on two models : VMS-Smag and SVV
- SVV is computationally efficient but VMS-Smag is more robust to a change of turbulent flow configuration
- Good performances on periodic hills with coefficients calibrated on HIT configuration
- LES results for wall-bounded flows better in regions where turbulence is close to HIT

Current limitations

- Uniform Cartesian grid
- Relatively low convergence order
- Calibration limited for turbulent flows around solid immersed bodies

Perspectives

Decoupled subgrid-scale modeling

- Depending on the subgrid-scale tensor term

$$R_{SGS} = \underbrace{\boxed{\overline{\boldsymbol{\omega} \otimes \mathbf{u}} - \overline{\boldsymbol{\omega}} \otimes \overline{\mathbf{u}}}}_{\text{Advection}} - \underbrace{\boxed{\overline{\mathbf{u} \otimes \boldsymbol{\omega}} + \overline{\mathbf{u}} \otimes \overline{\boldsymbol{\omega}}}}_{\text{Stretching}}$$

- Depending on the direction (Anisotropic SVV)

$$\partial_t \hat{\overline{\boldsymbol{\omega}}} = -\left(k_x^2(1/Re + \boxed{\nu_S(k_x, h_x)}) + k_y^2(1/Re + \boxed{\nu_S(k_y, h_y)}) + k_z^2(1/Re + \boxed{\nu_S(k_z, h_z)})\right)\hat{\overline{\boldsymbol{\omega}}}$$

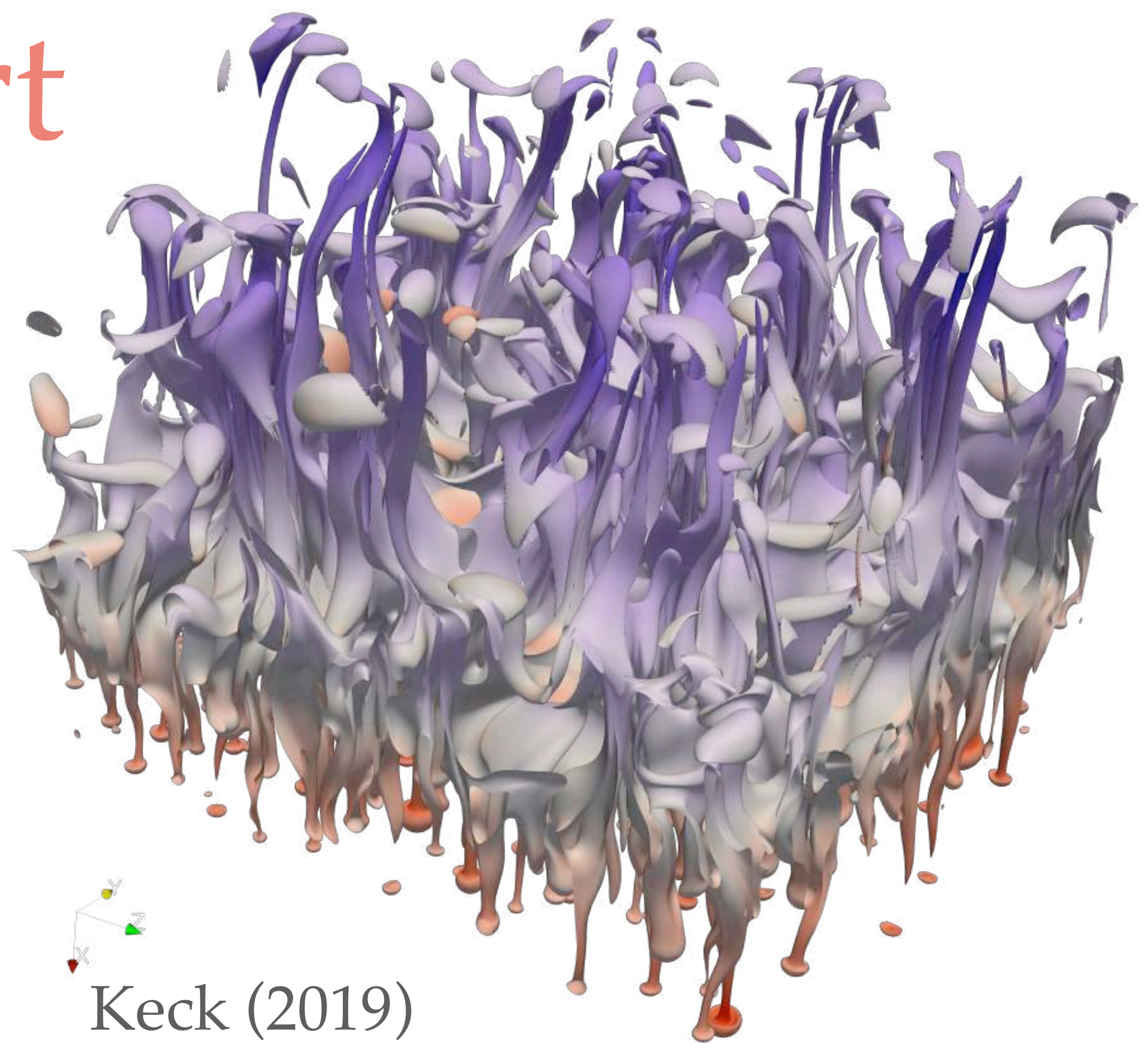
Conclusion

Multigrid simulations of scalar transport

Stability condition

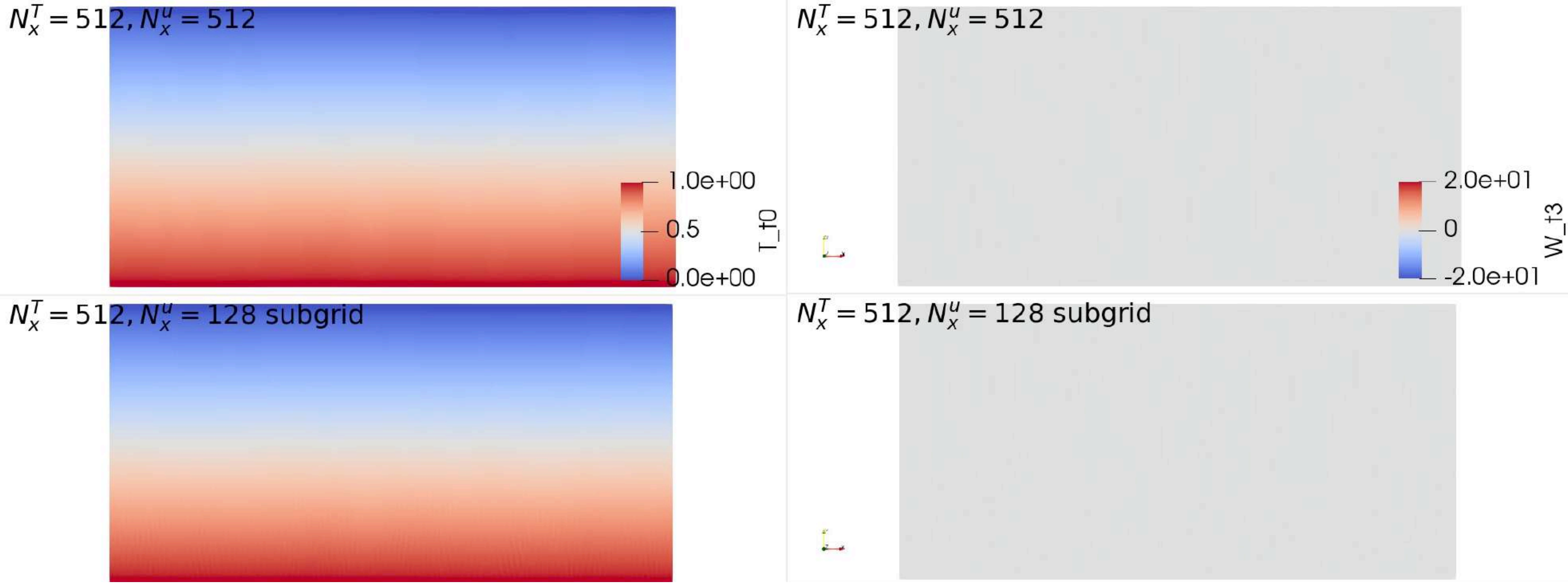
$$\Delta t = \frac{C_{LCFL}}{\|\nabla \mathbf{u}\|_\infty}$$

$$C_{LCFL} \leq 1$$



Keck (2019)

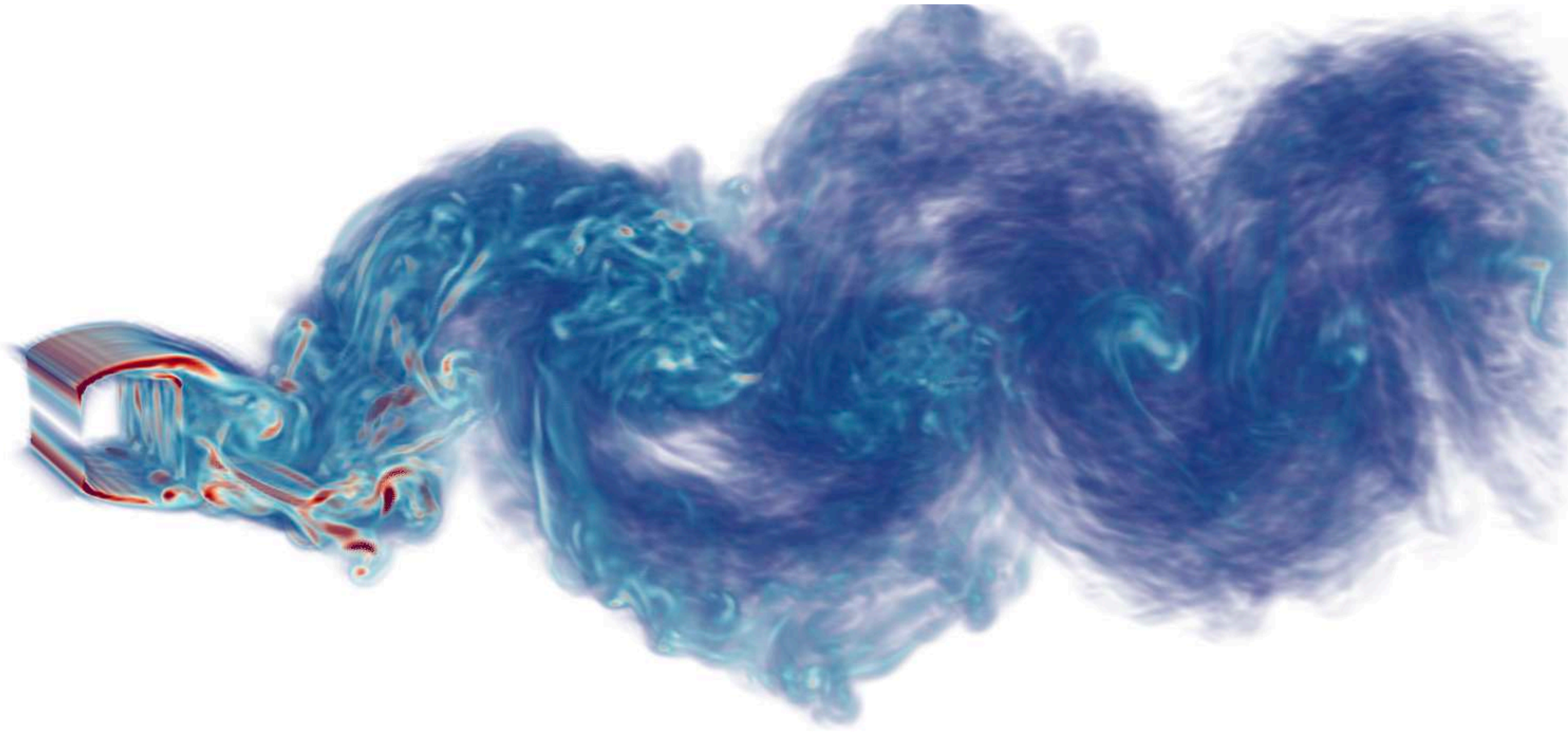
- Vortex methods advantaged in scalar transport cases where scalar diffusivity << fluid viscosity
  - Scalar transport naturally solved with particles
  - Possibility to use a fine grid for the scalar without decreasing the timestep and a coarse grid for the fluid
- Examples of applications
  - Sediment flows
  - Disease transmission, pollutant transport
  - Heat convection (Rayleigh-Benard)
- For active scalars : need for viscosity models to avoid numerical instabilities when interpolating fine scalar on coarse grid



Temperature

Vorticity

Thank you for your attention



Publications in journals

- **“Large-eddy simulations with remeshed vortex methods: An assessment and calibration of subgrid-scale models”**, M. de Crouy-Chanel, C. Mimeau, I. Mortazavi, A. Mariotti, M.V. Salvetti, Computers & Fluids, 2024

- Paper on periodic hills currently being written